

# Analysis day in memory of Mikael Passare

September 11, 2019



# **Organizers:**

Mats Andersson, Göteborg, matsa@chalmers.se Christer Kiselman, Uppsala, kiselman@it.uu.se Pavel Kurasov, Stockholm, kurasov@math.su.se

# ANALYSIS DAY IN MEMORY OF MIKAEL PASSARE

# **SEPTEMBER 11, 2019**

DEPT. OF MATHEMATICS, STOCKHOLM UNIVERSITY

# Program

12:00-13:00 Lunch at restaurant Kräftan

## Rum 14, Building 5, Kräftriket

- 13:15–14:00 Nils Dencker: The generic instability of differential operators
- 14:00–14:45 Håkan Hedenmalm: Planar orthogonal polynomials and arithmetic jellium.

## Coffee break

- 15:15–16:00 Alan Sola: Singularities of rational inner functions in higher dimensions
- 16:00-16:45 Andrzej Szulkin: A simple variational approach to weakly coupled elliptic systems

Visit to Norra begravningsplatsen

Dinner at Stallmästeregården



# Abstracts

## The generic instability of differential operators

Nils Dencker

Lund

nils.dencker@gmail.com

It came as a complete surprise when Hans Lewy in 1957 presented a non-vanishing complex vector field that cannot be solved anywhere. Actually, the vector field is the tangential Cauchy-Riemann operator on the boundary of a strictly pseudo-convex domain. Hörmander proved in 1960 that generically a linear PDE cannot be solved, unless it satisfies the bracket condition.

The bracket condition has many consequences for differential equations, such as the spectral stability, the stability of the Cauchy problem, the kernel and the range. For example, it is known that almost all three-dimensional vector fields only have the trivial kernel (the constants) and have unique ranges that determine the vector fields. In this talk we shall discuss the bracket condition and its generalizations.

# Planar orthogonal polynomials and arithmetic jellium. Håkan Hedenmalm KTH

haakanh@kth.se

This reports on joint work with Aron Wennman. We obtain a new asymptotic formula for the orthogonal polynomials associated with the random normal matrix model. Earlier work on related cases is by G. Szegő (1921), T. Carleman (1922), and P. K. Suetin (1969). We then consider a determinantal process with correlation kernel given in terms of a sum over the orthogonal polynomials with indices in an arithmetic progression, and call the resulting gas "arithmetic jellium".

## Singularities of rational inner functions in higher dimensions

### Alan Sola SU

#### sola@math.su.se

I will present recent joint work with Kelly Bickel (Bucknell) and James Pascoe (U Florida) concerning boundary behavior of rational inner functions in dimensions three and higher. On the analytic side, we use the critical integrability of the derivative of a rational inner function (RIF) of several variables to quantify the behavior of a RIF near its singularities, and on the geometric side we show that the unimodular level sets of a RIF convey information about its set of singularities. Our results, coupled with constructions of non-trivial RIF examples, demonstrate that much of the nice behavior previously seen in the two-variable case is lost in higher dimensions.

## A simple variational approach to weakly coupled elliptic systems

Andrzej Szulkin Stockholm andrzejs@math.su.se

The elliptic system of 2 equations

$$-\Delta u_i + \kappa_i u_i = \mu_i u_i^3 + 2\lambda u_i u_j^2, \quad i, j = 1, 2, \ i \neq j$$

where  $\Omega$  is a domain in  $\mathbb{R}^N$  has been extensively studied in dimensions  $N \leq 4$ . This system appears e.g. in 2-species physical problems (Bose-Einstein condensates with 2 hyperfine states) and in population dynamics. The condition  $\mu_i > 0$ signifies that the interaction of species (or particles) of the same kind is attractive while  $\lambda < 0$  signifies the repulsive interaction of species of different kind. Also various extensions (nonlinearities other than cubic, M instead of 2 equations) have been recently studied. In this talk we will be concerned with the system of M equations and nonlinearities which are not necessarily cubic. We introduce a general variational setting and then discuss existence and multiplicity of fully nontrivial (i.e.,  $u_i \neq 0$  for all i) solutions under different hypotheses on  $\Omega$  and the nonlinearity.

This is joint work with Mónica Clapp.