We look for nontrivial solutions to the semilinear problem

$$\nabla \times \nabla \times u = f(x, u) \text{ in } \mathbb{R}^3,$$

where $u : \mathbb{R}^3 \to \mathbb{R}^3$ and $f = \nabla_u F$. We give sufficient conditions on the nonlinearity which provide a ground state solution (i.e. a nontrivial solution with the minimal energy among all the nontrivial solutions) and infinitely many geometrically distinct solutions.

The growth and asymptotic behaviour of the nonlinearity are described by an N-function which allows us to consider other model problems than the classical power type or double-power type.

After building the proper function space where to look for solutions and showing its main characteristics, we develop an abstract critical point theory, providing results that we use, later on, to solve our equation, but that can be applied as well to a possibly larger class of problems.

The main difficulties are due to working in an unbounded domain and the infinite dimension of the kernel of $u \mapsto \nabla \times u$, i.e. the space of gradient vector fields. We overcome the former using a concentration-compactness argument.

This talk is based on a joint work with Andrzej Szulkin and Jarosław Mederski.