Spectral estimates and Ambartsumian-type theorems for quantum graphs.

Abstract: This thesis consists of four papers and deals with the spectral theory of quantum graphs. A quantum graph is a metric graph equipped with a self-adjoint Schrödinger operator acting on functions defined on the edges of the graph subject to certain vertex conditions.

In Paper I we establish a spectral estimate implying that the distance between the eigenvalues of a Laplace and a Schrödinger operator on the same graph is bounded by a constant depending only on the graph and the integral of the potential. We use this to generalize a geometric version of Ambartsumian's Theorem to the case of Schrödinger operators with standard vertex conditions.

In Paper II we extend the results of Paper I to more general vertex conditions but also provide explicit examples of quantum graphs that show that the results are not valid for all allowed vertex conditions.

In Paper III the zero sets of almost periodic functions are investigated, and it is shown that if two functions have zeros that are asymptotically close, they must coincide. This is relevant to the spectral theory of quantum graphs as the eigenvalues of a quantum graph are given by the zeros of a trigonometric polynomial, which is almost periodic.

In Paper IV we give a proof of the result in Paper III which does not rely on the theory of almost periodic functions and apply this to show that asymptotically isospectral quantum graphs are in fact isospectral. This allows us to generalize two uniqueness results in the spectral theory of quantum graphs: we show that if the spectrum of a Schrödinger operator with standard vertex conditions on a graph is equal to the spectrum of a Laplace operator on another graph then the potential must be zero, and we show that a metric graph with rationally independent edge-lengths is uniquely determined by the spectrum of a Schrödinger operator with standard vertex conditions on the graph.