Abstract

In this thesis we use the theory of algebraic operads to define a complete invariant of real and rational homotopy classes of maps of topological spaces and manifolds. More precisely let $f, g : M \to N$ be two smooth maps between manifolds M and N. To construct the invariant, we define an L_∞-structure on the space of linear maps $Hom_{\mathbb{R}}(H_*(M;\mathbb{R}), \pi_*(N)\otimes\mathbb{R})$ and a map

$$mc : Map_*(M,N) \to MC(Hom_{\mathbb{R}}(H_*(M;\mathbb{R}), \pi_*(N)\otimes\mathbb{R})),$$

from the set of based maps from M to N, to the set of Maurer-Cartan elements in $Hom_{\mathbb{R}}(H_*(M;\mathbb{R}), \pi_*(N)\otimes\mathbb{R})$. Then we show that the maps f and g are real (rational) homotopic if and only if $mc(f)$ is gauge equivalent to $mc(g)$, in this L_∞-algebra.

In the last part we show that in the real case, the map mc can be computed by integrating certain differential forms over certain subspaces of M. We also give a method to determine in certain cases, if the Maurer-Cartan elements $mc(f)$ and $mc(g)$ are gauge equivalent or not.