Chladni figures in circular membranes

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Abstract

If a drum membrane is sprinkled with fine sand and a speaker emits sound at certain frequencies sufficiently close, the sand will gather into special and interesting patterns. This thesis explores the mathematics to understand this phenomena.

We solve the *wave equation*

$$\frac{\partial^2}{\partial x^2}u(x,y,t) + \frac{\partial^2}{\partial y^2}u(x,y,t) = c^2\frac{\partial^2}{\partial t^2}u(x,y,t),$$

over the unit disc, which describes the motion of the membrane.

The solution we find is a series involving trigonometric functions and *Bessel functions*. In particular we see how the motion of the membrane can be described in terms of *eigenfunctions*, or fundamental shapes, vibrating at their own *eigenfrequency*. They, as a superposition, completely describe the motion of a drum.