Abstract:

For a compactly supported vector field $f = (f_1, \ldots, f_n)$ in $\mathbf{R}^n$ the ray transform If is defined by

$$If(x, \theta) = \sum_{j=1}^{n} \int_{\mathbf{R}} f_j(x + t\theta)\theta_j dt, \quad (x, \theta) \in \mathbf{R}^{2n},$$

and for a symmetric 2-tensor field $(f_{jk})_{j,k=1}^n$ one defines

$$If(x, \theta) = \sum_{j,k=1}^{n} \int_{\mathbf{R}} f_{jk}(x + t\theta)\theta_j \theta_k dt, \quad (x, \theta) \in \mathbf{R}^{2n}.$$

Since $If = 0$ if f is the gradient of a scalar function, one can at most recover the so-called solenoidal part ${}^s f$ of f from If. An analogous fact is true for second and higher rank tensor fields. Similarly, if the tensor field f is only defined in a bounded convex subset $\Omega \subset \mathbf{R}^n$, there is a natural definition of the solenoidal part ${}^s f$ of f relative to Ω. For tensor fields of arbitrary rank we give estimates for the norm of ${}^s f$ of the type

$$\|{}^s f\| \leq C \|If\|_{1/2},$$

where $\|\cdot\|$ is the L^2-norm and $\|\cdot\|_{1/2}$ is a Sobolev norm of order 1/2. The proof is based on a comparison of the Dirichlet integrals for the exterior and interior Dirichlet problems and a generalization of the Korn inequality to symmetric tensor fields of arbitrary rank. Weaker estimates for $\|{}^s f\|$ were given by L. Pestov and V. Sharafutdinov in 1988 for the more general case of the geodesic ray transform with respect to a Riemannian metric under a certain curvature condition on the metric. This is joint work with Vladimir Sharafutdinov.