Institute Mittag Leffler - workshop:

Herglotz-Nevanlinna functions and their applications

May 8-12, 2017

	Monday *Applications*	Tuesday *One variable*	Wednesday *Several variables*	Thursday *Applications*	Friday *Applications*
09:00-09:30	Introduction	Gesztesy	Ball	Figotin	Cherkaev
09:30-10:00					Ou
10:00-10:30					
10:30-11:00	Milton	Luger	Putinar	Sigal	open problem session
11:00-11:30					closure
11:30-12:00					
12:00-12:30	Lunch	Lunch	Lunch	Lunch	Lunch
12:30-13:00					
13:00-13:30					
13:30-14:00	Golden	Simon	Sigurdsson	Lindquist	
14:00-14:30					
14:30-15:00					
15:00-15:30	Gustafsson	Kurasov	Nedic	Sjöberg	
15:30-16:00		Eckhardt	Jonsson	Nordebo	
16:00-16:30					
16:30-17:00	Skaar	Engström		Cassier	
17:00-17:30				Welters	
17:30-18:00	Simon: Historic talk				
18:00-18:30			Workshopdinner		
18:30-19:00					
19:00-19:30					

Abstracts

Representation and interpolation theory for multivariable Herglotz classes

Ball Joseph A.

The classical Herglotz class (holomorphic functions mapping the unit disk to the right half plane) can be characterized via the Herglotz integral-formula representation. Furthermore, the Nevanlinna-Pick interpolation theorem gives an elegant matrix positive-definiteness test as to when there is a Herglotz function which takes on prescribed values at finitely many prescribed points in the unit disk. For the case of a holomorphic function on the polydisk, there are several possible notions of a multivariable Herglotz class of much more recent origin and development: (i) holomorphic functions on the polydisk with positive real part, (ii) holomorphic functions on the polydisk satisfying a von Neumann inequality (i.e., when extended to commuting operator arguments, the function maps a tuple of commuting Hilbert-space contraction operators to a contraction operator), and (iii) free noncommutative functions (in the sense of Kaliuzhnyi-Verbovetskyi–Vinnikov) mapping an arbitrary tuple of (not necessarily commuting) contraction operators to a contraction operator. We discuss analogues of the Herglotz integral representation and of the Nevanlinna-Pick interpolation theorem for these settings. Contrary to what one might expect, the results become increasingly definitive and elegant as one progresses from class (i) to class (ii) to class (iii). We also discuss versions of the results for the case where the polydisk is replaced by a poly-halfplane (e.g., a finite Cartesian product of the upper half plane with itself) and where the contraction-operator arguments are replaced by operator arguments having positive imaginary part.

Bounds on Herglotz functions and physical limits to broad-band passive cloaking in quasi-statics

Cassier Maxence

In this talk, we derive bounds on Herglotz functions which generalize those provided by A. Bernland, A. Luger, M. Gustafsson and D. Sjöberg. Then, we apply these bounds in the context of broadband passive cloaking in the quasi-static regime to negatively answer the following challenging question: is it possible to construct a passive cloaking device that cloaks an object over a whole frequency band? Our rigorous approach, although limited to quasi-statics, gives quantitative limitations on the cloaking effect over a finite frequency range by providing inequalities on the polarizabilty tensor associated with the cloaking device plus the cloaked object. We emphasize that our results hold for a cloak or object of any geometrical shape.

Joint work with Graeme W. Milton.

Herglotz functions in inverse homogenization

Cherkaev Elena

Inverse homogenization is a problem of deriving information about the microgeometry of a finely structured material from its known effective properties represented by a Herglotz function. The approach is based on the reconstruction of the matrix-valued measure in this representation relating the n-point correlation functions of the microstructure to the moments of the spectral measure of an operator depending on the composite's geometry. I show that the spectral measure that contains all information about the microstructure, can be uniquely recovered from the effective properties known in an interval of frequency. I will discuss identification of microstructural parameters from electromagnetic and viscoelastic effective parameters and present an extension of the approach to nonlinear materials. The resulting spectroscopic imaging method uses matrix Pade approximants that provide an efficient way to construct spectrally matched microstructures.

The inverse spectral problem for indefinite strings

Eckhardt Jonathan

One of the main objects in spectral theory for inhomogeneous vibrating strings is the so-called Weyl–Titchmarsh function; a Herglotz–Nevanlinna function which encodes all the spectral information. A classical result of M. G. Krein identifies the totality of all possible Weyl–Titchmarsh functions with the class of Stieltjes functions. I will review this result and show how it can be generalized to strings with indefinite mass distributions. This talk is based on joint work with A. Kostenko.

Spectral properties of Maxwell operator functions of Nevanlinna class

Engström Christian

Operator functions whose values are a Maxwell operator are used to describe metal-dielectric resonators and metamaterials.

In this talk, we assume that ϵ and μ depend on the frequency. Then, the Maxwell operator function is formally

$$\mathcal{A}(\omega) := \mathcal{M} - \omega\mathcal{F}(\omega), \quad \mathcal{M} := \begin{pmatrix} 0 & -i\mathrm{curl} \\ i\mathrm{curl} & 0 \end{pmatrix}, \quad \mathcal{F}(\omega) = \begin{pmatrix} \epsilon(x,\omega) & 0 \\ 0 & \mu(x,\omega) \end{pmatrix},$$

$\omega \in \mathcal{D}$, where ϵ and μ for fixed x are holomorphic in $\mathcal{D}$. The domain of $\mathcal{A}(\cdot)$ is chosen such that $\mathcal{M}$ is self-adjoint and we prove properties of the essential spectrum when $\mathcal{A}(\cdot)$ is a Nevanlinna function. Moreover, we prove that in some cases there will be a sequence of eigenvalues that have a limit point in the essential spectrum.

The talk is based on joint works with Heinz Langer, Axel Torshage, and Christiane Tretter.

Spectral Theory of Time Dispersive and Dissipative Systems

Figotin Alexander

We study linear time dispersive and dissipative systems. Very often such systems are not conservative and the standard spectral theory can not be applied. We develop a mathematically consistent framework allowing (i) to constructively determine if a given time dispersive system can be extended to a conservative one; (ii) to construct that very conservative system, which we show is essentially unique.

The developed methods feature an operator form generalization of the Bochner and the Herglotz?Nevanlinna theorems based upon the Naimark's theorem.

We illustrate the method by applying it to the spectral analysis of time dispersive dielectrics and the damped oscillator with retarded friction. In particular, we obtain a conservative extension of the Maxwell equations which is equivalent to the original Maxwell equations for a dispersive and lossy dielectric medium.

Based on joint work with J. Schenker.

Nevanlinna-Herglotz functions and some of their applications to spectral theory of differential and difference operators

Gesztesy Fritz

We intend to give a survey of (scalar, matrix-valued, and operator-valued) Nevanlinna-Herglotz functions (i.e., analytic maps from the open complex upper half-plane to itself) and some of their applications to spectral theory of differential and difference operators. Starting with elementary examples and basic properties of these functions, we hope to illustrate their use as Weyl-Titchmarsh functions and Dirichlet-to-Neumann maps in various situations (including ODEs and PDEs). While this talk primarily aims at nonspecialists in spectral theory, we also hope to present a few gems of interest for those working in spectral theory.

Herglotz functions in the mathematics of sea ice and other composites

Golden Kenneth M.

We will tour a broad range of applications of Herglotz functions and their integral representations in the context of homogenization for composite materials. In particular, we will focus on how Herglotz functions and these representations are being used in modeling sea ice, which exhibits composite structure over length scales ranging from millimeters to kilometers, and its role in the climate system. Application areas will include the effective propagation characteristics of quasistatic electromagnetic waves, advection diffusion phenomena, percolation, polycrystalline media, and ocean wave propagation in the marginal ice zone.

Herglotz functions, sum rules, and fundamental limitations on electromagnetic systems

Gustafsson Mats

An overview of the use of Herglotz functions to derive physical bounds on passive electromagnetic systems is presented. The bounds are derived by identification of a passive system, representation of the system with a Herglotz function, and using integral identities (sum rules) to limit the dynamic response with its low and high-frequency asymptotic expansion. These types of identities and limits are of great interest in many areas of physics and engineering. They also provide insight into the relationship between design parameters. We analyze and present physical bounds for; radar absorbers, temporal dispersion of metamaterials, extraordinary transmission, and antennas. We also compare the theoretical results with state of the art designs.

Properties of the transfer function in multi-dimensional systems

Jonsson Lars

A number of interesting results are closely connected to the time-passivity of physical systems. Time-passivity in linear, continuous and time-translation invariant system yields that the system can be described by a positive-real function, and there are numerous results on such system, including sum-rule based fundamental limitations and system representations with applications ranging from antennas to periodic structures. In this talk we discuss how passivity carry over to a multi-dimensional system, and what kind of constraints it implies on such systems.

Surgery of graphs and spectral gap: Titchmarsh-Weyl operator-function approach

Kurasov Pavel

Titchmarsh-Weyl M-functions prove to be an efficient tool to study spectral and inverse problems for the one-dimensional Schrödinger equation. In my talk I shall prove an explicit formula for the M-function in the case of compact finite metric graphs. It will be applied to study behaviour of the spectrum under surgery of graphs which we understand as gluing of two graphs together by identifying few vertices or cutting a graph by chopping through some of its vertices. It appears that a precise answer can be given in terms of the corresponding Titchmarsh-Weyl (matrix) functions of the two subgraphs, more precisely in terms of their negative spectral subspaces. We illustrate our findings by considering explicit examples.

Partially this a joint work with Sergey Naboko (S:t Petersburg).

The role of positive-real functions in systems theory

Lindquist Anders

In 1931 Otto Brune defined a positive-real function as a (rational) function analytic in the open right half-plane and mapping this domain into the closed open right half-plane. He observed that the impedance of any two-terminal electric network consisting of only resistors, inductors and capacitors had to be positive-real, but, for passive network synthesis, he wanted to know if any positive-real function could be realized in this way. This was finally proved in 1949 by Raoul Bott and Richard Duffin. After the invention of the "inerter" (the mechanical counterpart of the capacitor) in 2002 by Malcolm Smith, passive network synthesis can now also be applied to mechanical systems, as is done today in the suspension of formula-one racing cars.

In many other applications in systems theory, positive-real functions are instead analytic in the open unit disc and map to the closed open right half-plane. For example, when you talk in a mobile telephone, you solve a Carathodory extension problem every 30 milliseconds. In a seminal 1918 paper Friedrich Schur parameterized all meromorphic solutions to this problem in terms of what we today call the Schur parameters, and the solution used in the mobile phone is the one obtained by setting all "unknown" Schur parameters equal to zero. This solution is rational as required. In this talk I present a complete parameterization of all rational solutions of the same complexity and a convex-optimization approach for determining them. This parameterization is smooth and provides us with a "global-analysis approach", where one studies the whole family of solutions with smooth "tuning parameters" to adjust the solution.

This provides a powerful paradigm for smoothly parameterizing, comparing and shaping the solutions based on various additional design criteria and enables us to establish the smooth dependence of solutions on problem data. It can then be applied to other analytic interpolation problems. We present a similar theory for rational Nevanlinna-Pick interpolation and indicate how this procedure can be applied to high-resolution spectral estimation and robust control theory. If time allows, I shall also very briefly discuss applications to model reduction, system identification and image processing.

More about Herglotz-Nevanlinna functions in one variable

Luger Annemarie

In this talk we collect some more aspects of Herglotz-Nevanlinna functions that might be of interest. In particular, we focus on their role as Q-functions in classical extension theory and investigate how properties of the function are reflected in properties of the representing operator. Moreover, we will also have a short look on the class of generalised Nevanlinna functions.

A panorama of problems in material science and networks where Stieltjes and Herglotz functions emerge.

Milton Graeme

In the late 1970's and early 1980's, it was recognised that the effective dielectric constant of a composite was an analytic function of the component moduli, having positive imaginary part when the component moduli had positive imaginary part. These analytic properties, and the associated representation formulas, were found useful to deriving bounds on the response of composites. From a wider perspective many, but not all, of the bounds corresponded to known bounds on Stieltjes and Herglotz functions.

In two-dimensions one could find representative geometries: any function having the proper analytic properties could be associated with a sequentially layered laminate geometry. The analytic approach was extended to polycrystalline materials, to elasticity, to coupled problems such as thermoelectricity and piezoelectricity. For a two-dimensional polycrystalline materials, one can again find a correspondence with hierarchical laminate geometries, and consequently it is suggested that there may be alternative representation formulas that are more appropriate than the standard ones. Curiously tight bounds can be obtained by working in the time domain, rather than the frequency domain: by making measurements at specific times these allow one to recover, almost exactly, the volume fractions of the phases.

Recently, it has been discovered that the same sort of analyticity properties extend to the Dirichlet to Neumann map that governs the response of inhomogeneous bodies to electromagnetic, acoustic, or elastodynamic fields. This is exciting as it holds the promise of entirely new methods for solving the inverse problem of recovering the geometry inside the body from surface measurements. Analyticity properties have also proved useful to studying the electrical response of networks of (capacitive, inductive, or resistive) electrical networks, or to the dynamic response of mass-spring networks. In fact, within linear elasticity, one can completely characterize the response functions of mass-spring networks. There are other important problems where bounds on Stieltjes functions are important. Many physical moduli, such as complex dielectric constants are measured over an interval of frequencies. If this interval extended from zero to infinite frequencies one could apply the famous Kramer's Kronig relations to relate the real part of dielectric constant to the imaginary part: these are essentially Hilbert transforms. If however one only has measurements over a finite range, which is the case in practice, one can instead derive appropriate bounds.

This is/was joint work with Maxence Cassier, Karen Clark, David Eyre, Fernando Guevara Vasquez, Joseph Mantese, Ornella Mattei, Daniel Onofrei, Aaron Welters and many others. For further reading, see for example my book "The Theory of Composites" and our new book "Extending the Theory of Composites to Other Areas of Science".

On the class of boundary measures of Herglotz-Nevanlinna functions in several variables

Nedic Mitja

Herglotz-Nevanlinna functions are holomorphic functions defined in the poly-upper half-plane having non-negative imaginary part. These functions can be characterized via an integral representation involving a real constant, a linear part, a given integral kernel and a positive Borel measure.

In this talk, we investigate the class of measures that can appear in the aforementioned integral representation and present some properties of this class of measures. We will also highlight the differences between the class of measures in dimension one and in higher dimensions.

Passive approximation and optimization with applications for metamaterials

Nordebo Sven

A passive approximation problem is considered where the target function is an arbitrary complex valued continuous function defined on a closed interval of the real axis (the approximation domain). The approximating function is any Herglotz function which can be continuously extended to an open cover of the approximation domain, and the norm used is the usual max norm. This basic formulation leads to a unique greatest lower bound on the error norm, but in general, there is no unique solution. The problem of interest is to study the convergence properties of approximating Herglotz functions where the generating measures are defined by using finite B-spline expansions, and where the real part of the approximating functions are obtained by using the Hilbert transform. In practice, such approximations are readily obtained as the solution of a finite dimensional convex optimization problem. A convergence proof for the related B-spline approximations is sketched on, and discussed in this contribution. A typical physical application for this problem formulation is with the passive approximation of a linear system having metamaterial characteristics defined over a fixed finite bandwidth. A concrete example is considered based on optimal plasmonic resonances in small structures, where a dielectric medium is specified to have inductive properties (negative permittivity) over a finite bandwidth.

Joint work with Yevhen Ivanenko, Mats Gustafsson, Lars Jonsson, Annemarie Luger, Börje Nilsson

Application of Herglotz functions to the study of viscodynamics of porous media

Ou Yvonne

One important quantity in the study of porous media is the dynamic tortuosity, which characterizes the effective ebergy dissipation and the inertia coupling between the viscous fluid phase and the solid phase in a porous media. It plays the role of the the memory kernel in the generalized Biot equations, which are the governing equations of wave propagation in poroelastic media when the wave length is much longer than the length scale of the microstructure. In this talk, we will illustrate how the Herglotz function arises in the study of dynamic permeability, following the 1991 paper by Avellaneda and Torquato , and show how this result can be applied to the study of dynamic tortuosity and its Prony type approximation. Both the isotropic and anisotropic cases will be considered. We will show how the close tie between the Herglotz function and the Multipoint Pade (or rational) approximation is explored in this line of research.

Positive pluriharmonic functions in the ball

Putinar Mihai

This survey talk is aimed at isolating the obstacles encountered in the classification via integral representations of all complex analytic functions in the unit ball of $\mathbb{C}^d$ having non-negative real part. In particular we will discuss why the characterization of extreme rays in this convex cone of functions is challenging and still open. Theorems due to Aizenberg-Dautov, Forelli and Alexandrov will be incorporated in the discourse. A second part of the talk will focus on duality results (via Fantappie transform) and the relation via functional calculus to multivariate spectral theory.

On Quantum Decoherence

Sigal Michael

Decoherence is a key feature of Quantum Physics in which a quantum system looses its quantum properties (coherence) due to an interaction with a much larger system, called an environment. (Its analogue in the classical world is the loss of waves? coherence due to randomness of the source or an interaction with random scatterers.)

I present recent rigorous results on quantum decoherence for the standard model involving some simplified assumptions on the quantum system and the environment.

In particular, I will describe a careful mathematical formulation of the problem and the connection between decoherence and quantum resonances of the Liouvillean dynamics of the total system, leading to estimates of the decoherence and thermolization times. If time permits I will describe use of the rigorous renormalization group in this problem.

Boundary values of analytic functions and representation formulas for (pluri-)subharmonic functions.

Sigurdsson Ragnar

It the lecture I will discuss a few aspect of the theory of analytic functions of several variables in relation with the class of Herglotz-Nevanlinna functions. First I will discuss distribution limits of functions $x \mapsto f(x+iy)$, where f is analytic the imaginary part y tends to 0 within a convex cone. Examples of such functions are Fourier-Laplace transforms of distributions with support in a cone. Then I will discuss subharmonic functions and the Riesz representation formula in a half plane, which is a generalization of the integral representation formula for Herglotz-Nevanlinna functions.

Spectral Theory Sum Rules, Meromorphic Herglotz Functions and Large Deviations

Simon Barry

After defining the spectral theory of orthogonal polynomials on the unit circle (OPUC) and real line (OPRL), I'll describe Verblunsky's version of Szego's theorem as a sum rule for OPUC and the Killip–Simon sum rule for OPRL and their spectral consequences. Next I'll explain the original proof of Killip–Simon using representation theorems for meromorphic Herglotz functions. Finally I'll focus on recent work of Gambo, Nagel and Rouault who obtain the sum rules using large deviations for random matrices.

More Tales of our Forefathers

Simon Barry

This is not a mathematics talk but it is a talk for mathematicians. Too often, we think of historical mathematicians as only names assigned to theorems. With vignettes and anecdotes, I'll convince you they were also human beings and that, as the Chinese say, "May you live in interesting times" really is a curse. This talk will focus on the lives of 22 mathematicians including Gauss, Poincare, von Neumann, Loewner, Laundau and Noether. I gave an earlier talk "Tales of our Forefathers" several years ago. It is not assumed listeners heard that earlier talk. If you want to hear that talk, see http://www.math.caltech.edu/papers/bsimon/ForefathersVideo.mp4.

Physical Bounds and Convex Optimization Approximations for Electromagnetically Functional Surfaces

Sjöberg Daniel

An electromagnetically functional surface is typically realized as a periodic structure in the xy-plane with finite extent in the z-direction, providing functionality such as absorption, frequency or polarization selectivity, artificial magnetic conductivity etc. By controlling the microstructure or the temporal dispersion of the materials, the electromagnetic functionality can be controlled. The reflection or transmission coefficient of the structure can be associated with a Herglotz function, whose asymptotic values in the low and high frequency limit represent physical constraints such as the total thickness of the structure. Using the analyticity of Herglotz functions, sum rules and physical bounds restricting the electromagnetic functionality in terms of asymptotic values can be derived under some restrictions, which can be relaxed by considering convex optimization approximations based on an integral representation of Herglotz functions. The results can be used to characterize the maximum bandwidth obtainable for a certain thickness of the structure. This is joint work with Mats Gustafsson and Sven Nordebo.

Metamaterials - fundamental limitations, spatial dispersion, diamagnetism, and definitions of mu

Skaar Johannes

The electric susceptibility of electromagnetic media is usually assumed to obey the Kramers-Kronig relations (which means that it belongs to the Hardy space H^2 of the upper half-plane). This, together with passivity, gives fundamental limitations to the frequency dependence or dispersion. Similarly, it is natural to assume that the magnetic susceptibility belongs to H^2; however, the existence of diamagnetic media at zero frequency shows that this is not compatible with the usual passivity condition in general.

The resolution to this apparent paradox is spatial dispersion. To explore the space of possible effective parameters, we therefore will consider spatial dispersion in metamaterials. The importance of higher order multipoles will be stated. Also, five different definitions of the magnetic permeability, and some of their properties, will be discussed.

Analyticity of the Dirichlet-to-Neumann Map for Maxwell's Equations in Passive Composite Media

Welters Aaron

In this talk, I will discuss the analyticity properties of the electromagnetic Dirichlet-to-Neumann (DtN) map for the time-harmonic Maxwell's equations for passive linear multicomponent media. I will also discuss the connection of this map to Herglotz functions for isotropic and anisotropic multicomponent composites. The focus of the discussion will be on two different types of geometry, namely, layered media and bounded media (with Lipschitz domains). For these geometries I will derive the analyticity properties of the associated DtN map in terms of the transfer matrix for layered media and, for bounded media, using a variational formulation of the time-harmonic Maxwell's equations. This is joint work with Graeme Milton (Univ. of Utah) and Maxence Cassier (Univ. of Utah).