A semilinear Schrödinger equation with symmetric magnetic potential

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Abstract: We consider the magnetic Schrödinger equation

$$-(\nabla + iA(x))^2u = |u|^{p-2}u, \quad 2 < p \leq 2^* := 2N/(N-2) \ (N \geq 3)$$

where $A : \mathbb{R}^N \to \mathbb{R}^N$. The operator $\nabla + iA(x)$ appears in quantum mechanics of particles in an external magnetic field whose source is the magnetic potential A. We mainly focus our attention on the case $p = 2^*$ which is the critical exponent for the embedding of the Sobolev space $H^1(\mathbb{R}^N)$ into $L^p(\mathbb{R}^N)$. We discuss the existence of nontrivial solutions ($u \neq 0$) under the assumption that A is equivariant with respect to an action of a closed group $G \subset O(N)$ and we point out some connections to the equation

$$-\Delta u = |u|^{2^*-2}u, \quad x \in \mathbb{R}^N$$

related to the Yamabe problem. We also show that if G is "too large" ($G = SO(N)$), then the magnetic Schrödinger equation is equivalent to the non-magnetic one (the magnetic potential can be gauged away).