Extension property

The talk is based on a joint paper with John McCarthy.

Let D be a set in $\mathbb{C}^d$, and V be a subset of D, with no extra structure assumed. A function $f : V \to \mathbb{C}$ is said to be holomorphic if, for every point $a \in V$, there exists $t > 0$ and a holomorphic function F defined on the ball $B(a,t)$ in $\mathbb{C}^d$ such that F agrees with f on $V \cap B(a,t)$. Let $\mathcal{A}$ be an algebra of polynomials equipped with the sup-norm.

We say V has the $\mathcal{A}$ *extension property* if, for every f in $\mathcal{A}$, there exists $F \in \mathcal{O}(D)$, such that $F|_V = f$ and $||F|| = ||f||$, where both norms are the supremum of the modulus of the function over the appropriate set.

If D is pseudo-convex, and V is an analytic subvariety of D, it is a deep theorem of H. Cartan that every holomorphic function on V extends to a holomorphic function on D. The first result we know of norm-preserving extensions is due to W. Rudin, but with the extra hypothesis that the extension operator had algebraic structure. Agler and McCarthy studied the problem when D is the bidisc (see [1]). Recently, the result in this direction was obtained by Agler, Lykova and Young ([2]) for the symmetrized bidisc.

The purpose of my talk is to present a solution for a general class of domains (including polydiscs and strongly convex domains).

References:

[1] J. Agler and J.E. McCarthy, Norm preserving extensions of holomorphic functions from subvarieties of the bidisk, Ann. of Math. 157 (2003), no. 1, 289–312.

[2] J. Agler, Z. Lykova, and N.J. Young, Geodesics, retracts, and the norm-preserving extension property in the symmetrized bidisc., (2016) to appear in Memoirs of the American Mathematical Society