

F.O. de Paiva: Semilinear elliptic problem of Ambrosetti-Prodi type

Abstract: We prove an Ambrosetti-Prodi type result for the problem $-\Delta u = f(x, u) + t\varphi(x)$ in a bounded domain $\Omega \subset \mathbb{R}^N$, under the Neumann boundary condition $\frac{\partial u}{\partial \nu} = 0$, where u is in the Sobolev space $W^{2,p}(\Omega)$ ($p > N$). More precisely, we show that if $f(x, u) \rightarrow \infty$ as $u \rightarrow \infty$ and φ is nontrivial and non-negative, then there is $t_0 \in \mathbb{R}$ such that the problem has no solution, at least one solution or at least two solutions depending on whether $t > t_0$, $t = t_0$ or $t < t_0$.

The proof is based on a combination of upper and lower solutions method and the Leray-Schauder degree.