

CAUCHY INTEGRALS METHOD IN THE STUDY OF PERTURBATIONS OF OPERATORS

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Abstract

For a positive number α , we denote by Λ_α the space of all functions $f : \mathbb{D} \rightarrow \mathbb{C}$ which are analytic in the open unit disc $\mathbb{D} \subset \mathbb{C}$ and fulfil the Hölder condition of order α on \mathbb{D} . For $\alpha \in (0, 1)$, we show that if $f \in \Lambda_\alpha$, then f is an operator Hölder function of order α on the set of all linear contractions on a Hilbert space. Further, it is known that if $f \in \Lambda_1$, i.e that f is an analytic Lipschitz function on \mathbb{D} , then f need not be operator Lipschitz. We show that, if we add the property

$$\sup_{t \in \mathbb{R}} \iint_{\mathbb{D}} \frac{1 - |z|^2}{|e^{it} - z|^2} |f''(z)| dA(z) < \infty$$

where $z = x + iy$ and $dA = dx dy$, then f is operator Lipschitz. The two results are shown by tools from operator theory including the Spectral theorem and dilations of contractions.

We also solve a problem related to theory of dilations which was arisen on a mathematical question- and answer site. We show that if X, Y, Z are pairwise commuting operators on a Hilbert space with

1. $\|Z\| \leq 1$,
2. for any $z \in \mathbb{C}$ with $\|z\| \leq 1$,

$$\|X + zY\| \leq 1,$$

then $\|X + ZY\| \leq 1$ is false in general.