Workshop in Dynamical Systems

KTH May 11-13 2016

Abstracts

Magnus Aspenberg, Lund University

Semi-hyperbolic maps are rare

Abstract: Semi-hyperbolic maps were introduced by Carleson, Jones and Yoccoz in 1994 and is an interesting class of maps on the complex plane or the Riemann sphere, were (among other things) each critical point is non-recurrent. I will outline a recent result about these maps, proving that they have Lebesgue measure zero in the parameter space of rational maps of a fixed degree.

Michael Benedicks, KTH

Almost sure continuity along curves traversing the Mandelbrot set

Abstract: We study continuity properties of dynamical quantities while crossing the Mandelbrot set through typical smooth curves. In particular, we prove that for almost every parameter c_0 in the boundary of the Mandelbrot set M with respect of the harmonic measure and every smooth curve $\gamma : [-1, 1] \mapsto \mathbb{C}$ with the property that $c_0 = \gamma(0)$ there exists a set $\mathcal{A}_\gamma$ having 0 as a Lebesgue density point and such that $\lim_{x\to 0} HDim(J_{\gamma(x)}) = HDim(J_{c_0})$ for the Julia sets J_c. This is joint work with Jacek Graczyk.

Bassam Fayad, CNRS, Paris 7

Double exponential stability of elliptic fixed points in Hamiltonian dynamics : prevalence and optimality of the phenomenon

Abstract: The stability (or instability) of an equilibrium in Hamiltonian dynamics can be studied from various points of view : the classical Lyapunov stability or topological stability, the stability from a probabilistic point of view addressed by KAM theory, or the (even more classical!) effective stability. By effective stability, we mean the time spent by solutions that start close to the equilibrium in a neighborhood of the equilibrium. Combining KAM theory, Nekhoroshev theory and estimates of Normal Birkhoff forms we will explain why typically (in a very general measure theoretic and topological sense) equilibria as well as invariant quasi-periodic tori of Hamiltonian systems are double exponentially stable, in the sense that nearby solutions remain close to the equilibrium or invariant torus for an interval of time which is doubly exponentially large with respect to the inverse of the initial distance to the point or torus.

Using Herman synchronized diffusion, we show that the double-exponential stability is optimal, even for fixed points and tori with definite torsion.

Livio Flaminio , Université de Lille

Nil-sequences, multiplicative functions and asymptotic orthogonal powers

Abstract: An automorphism T of a probability space has asymptotic orthogonal powers if all ergodic joinings of the the product $T^r \otimes T^s$ converge to the product joining, when the relatively prime integers r and s go to infinity.

We show that the affine unipotent and ergodic diffeomorphisms of nilmanifolds have asymptotic orthogonal powers. Two consequences follow:

(i) The conjecture of Sarnak on the orthogonality of the Möbius function to deterministic systems is true for any automorphism measurably isomorphic to an affine unipotent and ergodic diffeomorphism of a nilmanifold.

(ii) In addition, for these automorphims, the above mentioned Sarnak conjecture "holds on small intervals".

(In collaboration with K. Fraczek, J. Kulaga-Przymus and M. Lemanczyk)

Michael Jakobson, University of Maryland, College Park

Ergodic and statistical properties of some attractors with countable Markov partitions

Abstract: We study certain piecewise smooth two-dimensional systems with countable Markov partitions. In particular we prove exponential decay of correlations by using several results of O.Sarig. Our approach is motivated by the original method of Anosov and Sinai from their 1967 paper.

Ángel Jorba, University of Barcelona

Fractalization of invariant curves in affine skew products of the plane.

Abstract: Quasi-periodically forced maps are discrete dynamical systems of the form

$$\left.\begin{array}{rcl} \tilde{x} & = & f(x,\theta,\mu), \\ \tilde{\theta} & = & \theta+\omega, \end{array}\right\} \tag{1}$$

where $x \in \mathbb{R}^n$, $\theta \in \mathbb{T}$, ω is Diophantine and μ is a real parameter. The map f is usually assumed to be of class C^r, $r \geq 1$. An invariant curve is a C^1 map $\theta \mapsto x(\theta)$ such that $f(x(\theta),\theta,\mu) = x(\theta+\omega)$. Assume that, for a given value of the parameter $\mu = \mu_0$, the map (1) has an attracting invariant curve, and that when μ goes from μ_0 to a critical value μ_1 this Lyapunov exponent goes to zero. We are interested in the possible behaviours of the invariant curve when μ approaches μ_1. In particular, we are interested in fractalization phenomena that might give rise to the appearance of a Strange Non-Chaotic Attractor.

To study this phenomenon we will focus on a simpler situation, given by the affine system

$$\left.\begin{array}{rcl} \tilde{x} & = & \mu A(\theta)x + b(\theta), \\ \tilde{\theta} & = & \theta + \omega, \end{array}\right\} \tag{2}$$

where ω is the golden mean, $x \in \mathbb{R}^2$ and, for each θ, $A(\theta)$ is a 2×2 real matrix and $b(\theta)$ a two-dimensional real vector. Moreover, we will assume that the corresponding linear system

$$\left.\begin{array}{rcl} \tilde{x} & = & \mu A(\theta)x, \\ \tilde{\theta} & = & \theta + \omega, \end{array}\right\}$$

is non-reducible due to a topological obstruction. A remarkable example of such a non-reducible system is given by

$$A(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

We will prove that (2) has an invariant curve that displays a fractalization process when μ goes to a critical value. (Joint with Nuria Fagella, Marc Jorba-Cuscó and Joan Carles Tatjer.)

Anders Karlsson, Uppsala/Geneva

Metric functionals, subadditive and multiplicative ergodic theorems

Abstract: Metric functionals are to metric spaces what linear functionals are to linear spaces. It is possible to develop some aspects of spectral theory using metrics instead of norms.These ideas have found an application in a rather general multiplicative ergodic theorem. This theorem applies to products of random matrices, bounded linear operators, holomorphic maps, topical operators and random walks on groups. A crucial ingredient is a new subadditive ergodic theorem that is a substantial refinement of the subadditive ergodic theorem of Kingman. Based on a joint work with S. Gouëzel.

Anatole Katok, Penn State

Two remarks on special orbits of area preserving maps near elliptic periodic points

Abstract: Our first remark is an elaboration and strengthening of an observation made by G.D Birkhoff in his classical treatise "Dynamical systems about existence of quasi-periodic orbits that appear as limits of elliptic periodic points of growing periods. In the general style of the time and his work Birkhoff was quite vague. What he had in mind was neither invariant curves, nor disconnected Aubrey-Mather sets but what in the modern language is called odometers. In a joint work we Kurt Vinhage we prove that for any odometer OD the set of $C^r, r > 3$ area preserving diffeomorphisms with an elliptic fixed point that have an invariant set with dynamics isomorphic to OD that appears naturally as a limit of elliptic period points is residual in C^2 topology. We will also have partial results on simultaneous existence on many odometers.

Our second remark is the following statement: Among C^∞ area preserving diffeomorphisms of a surface that have an elliptic periodic point those that have infinitely many ergodic components of positive Lebesgue measure are dense in C^r topology for any r. The proof is based on an application of the approximation-by-conjugation method.

Svetlana Katok, Penn State

Reduction theory for Fuchsian groups and coding of geodesics.

Abstract: I will discuss a method of coding of geodesics on quotients of the hyperbolic plane by Fuchsian groups using boundary maps and reduction theory. These maps are piecewise fractional-linear given by generators of the Fuchsian group, and the orbit of a point under the boundary map defines its boundary expansion. For compact surfaces they are generalizations of the Bowen-Series map, and for the modular surface are related to a family of (a, b)-continued fractions. For the natural extensions of the boundary maps Zagiers Reduction Theory Conjecture (RTC) holds: for the appropriate open sets of parameters they have attractors with finite rectangular structure to which (almost) every point is mapped after finitely many iterations.

The RTC is used for representing the geodesic flow as a special flow over a cross-section of reduced geodesics parametrized by the attractor. When a boundary expansion has a so-called dual, the coding sequences are obtained by juxtaposition of the boundary expansions of the end points of the corresponding geodesic, and the set of coding sequences is a sofic shift. This was proved for the modular group and generalizes for Fuchsian groups that satisfy the RTC. The talk is based on joint works with Ilie Ugarcovici.

Alexey Klimenko, Steklov Mathematical Institute, Moscow

Limit theorem for interval exchange maps

Abstract: A. Bufetov proved limit theorem for translation flows on flat surfaces (as well as for Vershik flows, their symbolic analogues). His result states that for a generic flow and a weakly Lipshitz function with zero average that does not belong to some linear subspace, the distribution of ergodic integral tends to the distibution of the finitely-additive cocycle corresponding to the second Lyapunov exponent of the flow.

I will discuss the similar result for interval exchange transformations. The difficulty here is that for a translation flow there is a flow along another foliation, hence one can use duality between the corresponding cocycles. For i.e.t.'s there are no such flow, so we need to develop some substitute.

Mikko Stenlund, University of Helsinki

Quasistatic dynamical systems

Abstract: We discuss an abstract framework for modeling quasistatic non-equilibrium processes arising in physics. After that we present limit results for a concrete model.
