Lectures about derived categories and cohomology of sheaves

Speaker: prof. emeritus Jan-Erik Björk

Four lectures - each 2 x 45 minutes - will present material based upon work by Grothendieck, Leray and Verdier. The lectures are foremost adressed to Phd-students and depending upon interest I am prepared to deliver some further lectures which, for example, treat homological algebra and more generally, cohomology attached to sheaves of modules on a ringed spaces.

The first two lectures are devoted to the construction of the derived category D(A) whose objects are chain complexes indexed by integers and whose individual objects belong to a given abelian category A. Thus, objects of D(A) are the same as in the abelian category of chain complexes C(A) but the abelian group of morphisms from one object X into another object Y taken in D(A) is more extensive as compared to the standard morphisms taken in $C(A)$. A notable point is that when $\phi\colon X \to Y$ is a *quasi-isomorphism*, i.e.the induced maps from individual cohomology groups $H^k(X) \to H^k(Y)$ are isomorphisms for every integer k, then ϕ is invertible and hence the chain complexes X and Y are isomorphic in the derived category. I shall expose the construction of $D(A)$ where my aim is to give a comprehensible background about derived categories. It will be based upon my paper entitled Derived Categories which appears in Springer Lecture Notes: Volume 1478 (1991) page 67-129.

An example of an abelian category is the family of abelian sheaves on a given topologicial space X. Given a sheaf complex with finite amplitude, i.e. a complex

$$0 \to X^0 \overset{d^0}{\to} X^1 \to \ldots \overset{d^{N-1}}{\to} X^N \to 0$$

where N is a positive integer, $\{X^\nu\}$ abelian sheaves on X and the differentials are ordinary sheaf morphisms, one constructs a long exact sequence of cohomology groups, which traditionally was attained from coverings of X and expressed via Cech Cohomology groups. A new and different method to grasp cohomology of complexes of sheaves was presented by Grothendieck in his article *Sur quelques points d'algèebre homologique*, published in Tohuku Math. Journal 1957 (vol 9 - page 119-221). The last two lectures will present Grothendieck's account of sheaves and their cohomology based upon a systematic use of derived categories. For example, when X is a topological space, one can regard the derived category $D^b(X)$ whose objects are complexes of abelian sheaves of finite amplitude on X. Thanks to the use of derived categories one gets a full understanding of "higher direct image sheaves" and many important results about induced maps of cohomology while one regards a map $F\colon X \to Y$ from one topological space into another. The last two lectures will present material from the chapter entitled *Sheaf Theory* - page 421-454) in my book Analytic $\mathcal{D}$-modules, published in Kluwer Math. Library Series 1991.

Time and place

The first lecture is delivered January 16 - 2016 from 15^{15} to 17^{00} in room 306 at the department of mathematics of SU. The later lectures take place at the same place and time during the subsequent three monday afternoons.