Analysis Day

Stockholm University, September 28, 2015

On the occasion of the conferment of the new

Honorary Doctor Heinz Langer

The lectures take place in room 306, house 6, kräftriket.

09:30 - 09:40	Opening
09:40 - 10:15	**Aad Dijksma** *Finite codimensional Compressions*
10:20 - 10:55	**Marco Marletta** *The finite section method for non-selfadjoint Jacobi matrices, Schrödinger operators and some pencils*
11:00 - 11:30	Coffee break
11:30 - 12:05	**Matthias Langer** *Some aspects of* $\mathcal{N}_\kappa^{(\infty)}$*-functions*
12:10 - 12:45	**Jussi Behrndt** *Q-functions, Weyl functions and Dirichlet-to-Neumann maps*
12:50 - 14:10	Lunch
14:10 - 14:45	**Harald Woracek** *The order problem for Hamburger Hamiltonians*
14:50 - 15:25	**Gerald Teschl** *Spectral asymptotics for canonical systems*
15:30 - 16:00	Coffee break
16:00 - 16:35	**Christiane Tretter** *Spectral theory and applications - a tribute to Heinz Langer*
17:00	Get together in the library
18:30	Dinner

Abstracts

Finite codimensional compressions
by Aad Dijksma (Groningen, The Netherlands)

Let $\mathcal{H}$ be a Hilbert space, let $\mathcal{G}$ be a subspace of $\mathcal{H}$ with $\operatorname{codim}\mathcal{G} < \infty$, let $P_{\mathcal{G}}$ be the orthogonal projection in $\mathcal{H}$ onto $\mathcal{G}$, let T be an operator or linear relation in $\mathcal{H}$ and let $T_0 = P_{\mathcal{G}}T|_{\mathcal{G}\cap\operatorname{dom}T}$ be the *finite codimensional compression* of T in $\mathcal{H}$ to $\mathcal{G}$.

W. Stenger (1968) proved: *If T is a self-adjoint operator in $\mathcal{H}$, then T_0 is a self-adjoint operator in $\mathcal{G}$.*

M.A. Nudelman (2011) proved: *If T is a maximal dissipative operator in $\mathcal{H}$, then T_0 is a maximal dissipative operator in $\mathcal{G}$.*

The operators here are all densely defined, because recall: *A self-adjoint or maximal dissipative linear relation in a Hilbert space is an operator if and only if it is densely defined.*

In the lecture we briefly discuss the converse of these results and analogous statements for linear relations in Krein spaces. The emphasis of the lecture however is on additional properties that a finite codimensional compression T_0 of T may or may not inherit from T.

The lecture is based on joint work in progress with Tomas Azizov (Voronezh, Russia) and Branko Curgus (Bellingham, WA, USA).

Q-functions, Weyl functions and Dirichlet-to-Neumann maps
by Jussi Behrndt (TU Graz, Austria)

In the extension theory of symmetric operators the Q-function introduced and studied in works by M.G. Krein and H. Langer plays a fundamental role. In particular, if the underlying symmetric operator is simple or completely non-selfadjoint then the spectral properties of all selfadjoint extensions are completely encoded in the Q-function. Any Q-function can be viewed as a Weyl function corresponding to a so-called boundary triple, and vice versa and Weyl function of a boundary triple can be interpreted as a Q-function, so that, roughly speaking, the notions of Q-functions and Weyl functions coincide. In this talk we discuss how the concept of Q-functions and Weyl functions may be generalized such that it can be used directly in applications involving PDEs. More precisely, we show that the Dirichlet-to-Neumann map in the analysis of second order elliptic differential operators can be viewed as a generalized Q or Weyl-function. In particular, these observations lead to spectral characterizations of second order PDE's via Dirichlet-to-Neumann maps. This talk is based on joint work with Jonathan Rohleder.

The finite section method for non-selfadjoint Jacobi matrices, Schrödinger operators and some pencils
by Marco Marletta (Cardiff University, UK)

In this talk we will review recent and ongoing work with Sabine Bögli and Sergey Naboko.

We will show that for certain classes of dissipative Schrödinger operators in dimension $d \leq 3$, the finite section method never fails to approximate any part of the essential spectrum. For Jacobi matrices this is also true under weaker hypotheses and the proof, which is easier in this case, gives an indication of how the other cases may be treated.

We consider the question of spectral pollution. Using the new concept of essential numerical range for pencils, we obtain smaller enclosures for sets of spectral pollution than have hitherto been possible.

Some aspects of $\mathcal{N}_\kappa^{(\infty)}$-functions
by Matthias Langer (University of Strathclyde, UK)

The class $\mathcal{N}_\kappa^{(\infty)}$ of generalised Nevanlinna functions that have a generalised pole of non-positive type only at infinity has attracted a lot of interest recently. In this talk I will present distributional representations of such functions and their connection with indefinite canonical systems.

Spectral asymptotics for canonical systems
by Gerald Teschl (University of Vienna, Austria)

Based on continuity properties of the de Branges correspondence, we develop a new approach to study the high-energy behavior of Weyl–Titchmarsh and spectral functions of 2×2 first order canonical systems. Our results improve several classical results and solve open problems posed by previous authors. Furthermore, they are applied to radial Dirac and radial Schrödinger operators as well as to Krein strings and generalized indefinite strings.

Spectral theory and applications - a tribute to Heinz Langer
by Christiane Tretter (University of Berne, Switzerland)

This talk will highlight some of my recent joint work with Heinz Langer and different collaborators. In particular, I will focus on spectral problems relating to highly topical applications in physics and engineering.

The order problem for Hamburger Hamiltonians
by Harald Woracek (TU Wien, Austria)

Let H be a trace-normed positive semidefinite 2-dimensional Hamiltonian on a finite interval $[0, L]$, and let $W(x, z)$, $x \in [0, L]$, be the fundamental solution of the canonical system with Hamiltonian H. Then the entries $w_{ij}(L, z)$ of $W(L, z)$ are entire functions of the Cartwright class. All the four functions have the same exponential type which is given by the Krein-de Branges formula

$$\text{exponential type of } w_{ij}(L, .) = \int_0^L \sqrt{\det H(x)}\, dx.$$

If $\det H(x) = 0$, $x \in [0, L]$ a.e., the order of $w_{ij}(L, .)$ may be less than 1. It is known that the functions $w_{ij}(L, .)$, $i, j = 1, 2$, have the same order, call it $\rho(H)$, and that for arbitrary $\rho \in [0, 1]$ there exist Hamiltonians H with $\det H = 0$ a.e. and $\rho(H) = \rho$. A problem that arises naturally is to determine or estimate $\rho(H)$ for given H.

The – probably – first result dealing with growth properties different from exponential type is due to M.S.Livšic back in 1939. Livšic shows that the order of the four entire functions involved in the Nevanlinna parameterisation corresponding to an indeterminate (Hamburger-) moment sequence $(s_n)_{n=0}^{\infty}$ is not less than the order of the entire function $\sum_{n=0}^{\infty} \frac{z^{2n}}{s_{2n}}$. Whether or not these orders always coincide remained an open problem ever since the work of Livšic.

We consider Hamiltonians corresponding to indeterminate power moment problems, i.e., which consist of a sequence of indivisible intervals accumulating only at L. Following I.S.Kac we call them Hamburger Hamiltonians. It is shown in recent work of C.Berg and R.Szwarc that for such Hamiltonians $\rho(H)$ can be computed in terms of the coefficients of orthogonal polynomials (all coefficients are involved). Clearly, this formula is nearly impossible to apply since the orthogonal polynomials can be fully computed only in some sporadic cases when knowledge about special functions can be employed.

In this talk we present:

1. An upper estimate for $\rho(H)$ which is explicit in terms of the Hamiltonian (lengths and angle of the indivisible intervals).

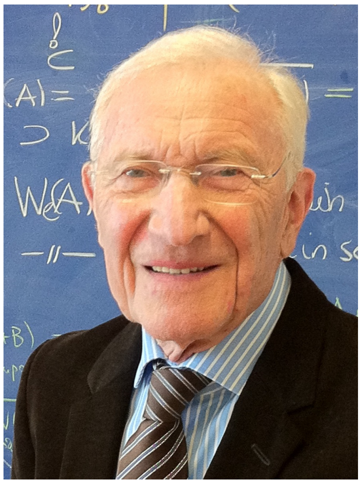
2. A lower estimate which can be deduced from the Berg-Szwarc formula (or, probably simpler, be proven directly).

3. A regularity condition on the lengths and angles which ensures that these estimates coincide and hence establish a formula for $\rho(H)$ which is explicit in terms of lengths and angles; the message is that the faster lengths decay and the smoother angles vary, the smaller $\rho(H)$ will be.

4. Examples which show that dropping regularity can lead to situations where both estimates do not coincide with $\rho(H)$.

In particular, we show that in Livšic's Theorem equality does not always hold.
Joint work with *Raphael Pruckner* and *Roman Romanov*

From the "installationsskrift":



Prof. Em. Dr. Heinz Langer is an inspiring mathematician and influential teacher. He was educated and worked many years as professor at the Techn. Univ. Dresden (then GDR) before moving to Vienna Univ. of Technology.
Heinz is a world leading expert in spectral theory, a branch of modern mathematical analysis with applications in many fields, where he has moved boundaries and opened new areas of research. A distinctive quality is his lively interest in the results and ideas of others, regardless of origin or position of the author. This has led to numerous connections, both in the East and the West.

He has been a frequent visitor to Sweden during many years and the current analysis group at the department of mathematics is strongly influenced in many ways by both his work and his personality.