

Multiple solutions to a nonlinear Choquard equation

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We consider the generalized Choquard equation

$$-\Delta u + V(x)u = (|x|^{-\alpha} * |u|^p)|u|^{p-2}u, \quad u \in H_0^1(\Omega),$$

where Ω is an exterior domain in \mathbb{R}^N , $N \geq 3$, V is a scalar potential, $\alpha \in (0, N)$ and $p \in [2, \frac{2N-\alpha}{N-2})$.

The special case

$$-\Delta u + u = (|x|^{-1} * |u|^2)u, \quad u \in H^1(\mathbb{R}^3),$$

is commonly referred to as the stationary Choquard equation. It arises in an approximation to Hartree-Fock theory for a one component plasma, and has many interesting applications in the quantum theory of large systems of non-relativistic bosonic atoms and molecules. This equation was also introduced by Penrose in his discussion on the self-gravitational collapse of a quantum mechanical wave-function. In this context it is usually called the Schrödinger-Newton equation.

We shall present some multiplicity results for this problem, recently obtained with Dora Salazar (Universidad Nacional Autónoma de México).