# Combinatorial Methods in Complex Analysis


## Per Alexandersson


Academic dissertation for the Degree of Doctor of Philosophy in Mathematics at Stockholm University to be publicly defended on Thursday 30 May 2013 at 13:00 in Lecture hall 14, House 5, Kräftriket, Roslagsvägen 101.


## Abstract

The theme of this thesis is combinatorics, complex analysis and algebraic geometry. The thesis consists of six articles divided into four parts.

**Part A:** *Spectral properties of the Schrödinger equation*

This part consists of Papers I-II, where we study a univariate Schrödinger equation with a complex polynomial potential. We prove that the set of polynomial potentials that admit solutions to the Schrödingerequation is connected, under certain boundary conditions. We also study a similar result for even polynomial potentials, where a similar result is obtained.

**Part B:** *Graph monomials and sums of squares*

In this part, consisting of Paper III, we study natural bases for the space of homogeneous, symmetric and translation-invariant polynomials in terms of multigraphs. We find all multigraphs with at most six edges that give rise to non-negative polynomials, and which of these that can be expressed as a sum of squares. Such polynomials appear naturally in connection to expressing certain non-negative polynomials as sums of squares.

**Part C:** *Eigenvalue asymptotics of banded Toeplitz matrices*

This part consists of Papers IV-V. We give a new and generalized proof of a theorem by P. Schmidt and F. Spitzer concerning asymptotics of eigenvalues of Toeplitz matrices. We also generalize the notion of eigenvalues to rectangular matrices, and partially prove the a multivariate analogue of the above.

**Part D:** *Stretched Schur polynomials*

This part consists of Paper VI, where we give a combinatorial proof that certain sequences of skew Schur polynomials satisfy linear recurrences with polynomial coefficients.



**Keywords:** *combinatorics, Schrödinger equation, Toeplitz matrix, sums of squares, Schur polynomials.*

Stockholm 2013
http://urn.kb.se/resolve?urn=urn:nbn:se:su:diva-88808

ISBN 978-91-7447-684-2




## Department of Mathematics

Stockholm University, 106 91 Stockholm