Workshop i dynamiska system

KTH 18–19 april 2013

Abstracts

Magnus Aspenberg, Lund

Collet-Eckmann and Misiurewicz

Abstract: Uniformly expanding dynamical systems in the complex setting (also called hyperbolic maps) is believed to be generic (open and dense) in the parameter space. This was conjectured already by P. Fatou in the beginning of the 20th century and still unsolved. Uniformly expanding here means on the Julia set, and hence in particular there cannot be any critical points on it. To relax this strong condition other non-uniformly expanding conditions were introduced, for instance the so called Collet-Eckmann condition. This means that there are critical points on the Julia set but the derivative of iterates of the critical value(s) grow exponentially. Collet-Eckmann maps have positive measure in the parameter space for rational functions, and moreover one has to admit that the critical orbit(s) approach the critical point(s). Critically non-recurrent maps, such as Misiurewicz maps, are a special type of Collet-Eckmann maps but they have measure zero. I will summarize the measure theoretic picture for these maps and also set them in connection to hyperbolic maps.

Michael Björklund, ETH

Coupling estimates and effective mixing of all orders

Abstract: We shall discuss a general method to prove quantitative results on (uniform) mixing of all orders for homogeneous (finite volume) actions of semisimple Lie groups (and adelic groups). If time permits, we will also discuss some applications to counting points on lattice orbits and geometry of lattices in Lie groups.

Joint work with M. Einsiedler (ETH) and A. Gorodnik (Bristol).

Neil Dobbs, Helsinki

Ergodic properties in transcendental dynamics

Abstract: General results will be discussed concerning typical long-term behaviour for a class of one-dimensional complex transcendental dynamical systems, before focussing in on dynamics of the exponential family $z \mapsto \lambda e^z$. We shall present results both for classes of maps and for parameter space.

Jordi-Lluis Figueras, Uppsala

Computer-assisted techniques for the verication of the Chebyshev property of Abelian integrals

Abstract: We develop techniques for the verication of the Chebyshev property of Abelian integrals. These techniques are a combination of theoretical results, analysis of asymptotic behavior of Wronskians, and rigorous computations based on interval arithmetic. We apply this approach to tackle a conjecture formulated by Dumortier and Roussarie in [Birth of canard cycles, Discrete Contin. Dyn. Syst. 2 (2009), 723781], which we are able to prove for $q \leq 2$

Denis Gaydashev, Uppsala

Renormalization and rigidity in one- and two-dimensional dynamics

Abstract: We will outline how renormalization can be used to construct conjugacy classes of one- and two-dimensional maps, and how regular these conjugacies are in the one-dimensional, two-dimensional dissipative and two-dimensional conservative settings.

This is a survey talk.

Tomas Persson, Lund

Borel-Cantelli lemmata in hyperbolic dynamics

Abstract: This is a joint work with Nicolai Haydn, Matthew Nicol and Sandro Vaienti. I will talk about how to obtain dynamical BorellCantelli lemmata in hyperbolic dynamics from assumptions on decay of correlations and return times.

Jörg Schmeling, Lund

Multifractal analysis of some multiple ergodic average - the invariant spectrum

Abstract: Let (X, T) be a topological dynamical system where T is a continuous map on a compact metric space X. Fürstenberg had initiated the study of the *multiple ergodic average*:

$$\frac{1}{n} \sum_{k=1}^{n} f_1(T^k x) f_2(T^{2k} x) \cdots f_s(T^{sk} x) \tag{1}$$

where $f_1, \cdots, f_s$ are s continuous functions on X with $s \geq 2$ when he proved the existence of arithmetic sequences of arbitrary length amongst sets of integers with positive density. Later on, the research of such a kind of average has attributed a lot of attentions.

We study the multiple ergodic averages

$$\frac{1}{n} \sum_{k=1}^{n} \varphi(x_k, x_{kq}, \cdots, x_{kq^{\ell-1}})$$

on the symbolic space $\Sigma_m = \{0, 1, \cdots, m-1\}^{\mathbb{N}^*}$ where $m \geq 2, \ell \geq 2, q \geq 2$ are integers. In his talk Ai-Hua Fan gives a complete solution to the problem of multifractal analysis of the limit of the multiple ergodic averages.

We will continue by considering the invariant part of the mul- tifractal level sets, i.e. we will study the maximal dimension of an invariant or multiple mixing measure supported on these level sets. Here many new interesting phenomena occur. In general there will be no invariant measure with the same dimension as the level sets. Moreover the invariant and the mixing spectra differ. On the other hand we will point on some connections to probability theory (von Mise statistics), ergodic optimization of multiple integrals and also indicate some new phase transition phenomena.

This is joint work with Ai-Hua FAN, and Meng WU.

Daniel Schnellmann, ENS, Paris

Law of iterated logarithm for the turning point in one-parameter families of tent maps

Abstract: In this talk we show that for almost every map in a non-degenerate one-parameter family of tent mapsthe Birkhoff sum of suitable observables along the forward orbit of the turning point satisfies the law of iterated logarithm. In fact we will prove an almost sure invariance principle for the Birkhoff sum (as a function on the parameter space). This is a stronger property than the law of iterated logarithm.

Warwick Tucker, Uppsala

A rigorous study of short periodic orbits for the Lorenz system

Abstract: In this talk, we describe a method for finding all short cycles embedded in a singular-hyperbolic attractor. It is shown that the problem of existence of periodic orbits can be studied rigorously by means of a symbolic dynamics approach combined with interval methods. Symbolic dynamics is used to find approximate initial positions of periodic points, and interval operators are used to prove the existence of periodic orbits in a neighborhood of the computer-generated approximate solution. As an example the Lorenz system is studied; all 8798 periodic orbits of the Poincare map with the period 16 or less are found.

This is joint work with Zbigniew Galias.

Denis Volk, KTH

Dynamics of Piecewise Translations

Abstract: Piecewise translations are piecewise continuous maps such that on every branch of continuity, they are just parallel translations. To get an idea what they look like in dim=1, begin with an interval exchange, and then let the images of intervals overlap. Since the original paper of 1995, there were very few general results in this area. I will present the recent progress in dim=1 and even more recent in dim¿1.

Anders Öberg, Uppsala

The Kusuoka invariant measure and energy Laplacians on fractal sets