A course for doctoral and advanced undergraduate students

Spectral Theory:

an Introduction to Mathematical Methods of Quantum Mechanics

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Mathematics, SU

7.5 points, Autumn semester 2012

The course will be devoted mainly to the spectral theory of differential operators with focus on applications in quantum mechanics. Differential operators can be seen as linear transformations in infinite dimensional spaces and spectral theory can be considered as an attempt to understand diagonalization of infinite size matrices corresponding to such transformations. As a byproduct one obtains expansions in terms of eigenfunctions generalizing classical Fourier analysis. The role of spectral theory is not limited to proving the spectral theorem for self-adjoint operators, but forms the foundation of quantum mechanics, where physical systems are described precisely by self-adjoint operators acting in infinite dimensional Hilbert spaces. Such operators similar to Hermitian matrices have real spectrum ensuring that physical observables for different states give real numbers, which can be measured in experiments. Development of spectral theory can be successfully carried out only if ideas from both mathematics and physics are used simultaneously.

The main goal of the course is to give an introduction to spectral theory of self-adjoint operators with focus on applications in quantum mechanics. In particular the following topics will be covered:

- Compact linear operators
- Unbounded liner operators: symmetric and self-adjoint operators
- Extension theory for symmetric operators
- Mathematical formulation of quantum mechanics
- Perturbation theory
- Self-adjoint extensions and quantum mechanics
- Spectral properties of different self-adjoint extensions

Prerequisites: basic knowledge of functional analysis and operator theory, Sturm-Liouville problems, ordinary differential equations.

First meeting: **September 3, 2012**, 13.15, room 306, house 6, Kräftriket (Albano).

If you are interested in attending the course, please send an e-mail to Pavel Kurasov pak@math.su.se.

This course can be considered as a natural continuation of the standard courses of Functional Analysis, Fourier Series and Differential Equations (both ordinary and partial) and is oriented towards advanced graduate and postgraduate students.

There will be one two-hour lecture every week. The main idea of the course is to introduce students into concrete problems from quantum mechanics related to operator theory. Therefore a part of every lecture will be devoted to realistic differential operators appearing in different problems of modern mathematical physics.

In the middle of the course the students will be asked to carry out research oriented projects working in groups or individually. The last couple of lectures will be devoted to presentation of these projects serving as examination for the course for those who are interested in carrying out research projects. In case of interest it will be possible to continue successful projects and prepare scientific reports on the obtained results.

Literature: There is no monograph which reflects the contents of the course, but different chapters will be taken from the following books:

1. N. Akhiezer and I. Glazman, Theory of Linear Operators in Hilbert Space (any edition)

2. L. Faddeev and O.Yakubovskii, Lectures on Quantum Mechanics for Mathematics Students, AMS, 2009.

3. M. Reed and B. Simon, Methods of Modern Mathematical Physics, vol II: Fourier Analysis, Self-adjointness (any edition).

4. L. Tahtajan, Quantum mechanics for mathematicians, AMS, 2008.

5. G. Teschl, Mathematical Methods in Quantum Mechanics; With Applications to Schrödinger Operators, AMS, 2009
(can be downloaded from http://www.mat.univie.ac.at/ gerald/)