ON TOEPLITZ PRODUCTS ON BERGMAN SPACE AND TWO-WEIGHTED INEQUALITIES

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ABSTRACT. In the early 90's, D. Sarason posed conjectures on the characterization of the boundedness of Toeplitz products on Hardy and Bergman spaces. The Hardy space case attracted much attention because of its close relation to the joint A_2 conjecture for the famous two-weight problem for the Hilbert transform in Real Analysis, pointed out by Cruz-Uribe in [1], but both conjectures, the Sarason conjecture for Toeplitz products on Hardy space and the joint A_2 conjecture, were shown to be false by F. Nazarov around 2000 [3]. The Hardy space case has been the subject of intense activity since then, with the work of Nazarov, Treil, Volberg, Lacey, Sawyer and many others, e.g. [4], [2].

The Bergman space case of Sarason's conjecture is still open, and is likewise connected to two-weighted inequalities on Bergman space.

In the talk, I will present some dyadic models for Toeplitz products on Bergman space and two-weighted operators, give necessary and sufficient conditions in this case, and also comment on necessary and sufficient conditions for the Toeplitz products.

This is joint work with Alexandru Aleman and Maria Carmen Reguera (both Lund).

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