**DISPUTATION I MATEMATIK**
**Farid Bozorgnia**

skall disputera på avhandlingen

**Numerical Algorithms for Free Boundary Problems of Obstacle Types**

torsdagen den 20 augusti 2009 kl. 14.00 i sal H1, KTH, Teknikringen 33, 2 tr. Till opponent har utsetts *Assistant Professor Adam Oberman*, Simon Fraser University, Burnaby, Canada.


***Abstract of the thesis***

This thesis consists of four scientific papers concerning numerical methods for certain free boundary problems. The papers include mathematical analysis of different approximations of the problems and the description of numerical implementation along with numerical results.

Paper I deals with a free boundary problem that appears in biology modelling. Two novel iterative methods for a class of population models of competitive type are introduced. The numerical approximations are related to the positive solution as the competitive rate tends to infinity. Furthermore, the first method is applied to an optimal partition problem.

In Paper II we study perturbation of the following free boundary problem

$$\begin{cases} \Delta u_i = \lambda^+ \chi_{\{u_i>0\}} - \lambda^- \chi_{\{u_i<0\}} & \text{in } \Omega, \\ u_i = g_i & \text{on } \partial\Omega. \end{cases} \tag{1}$$

We perturb the data in the right-hand side of the two-phase problem and the boundary values $g$. The main result of the paper is a proof of the continuity and differentiability of the solution with respect to the coefficients. Also the stability of the solution and the free boundary with respect to perturbation in coefficients and boundary values, is shown.

In Paper III different numerical approximations for a two-phase membrane problem are discussed. In the first method a new iterative method with different examples is presented. We also study the regularization method and give an a posteriori error estimate which is needed for the implementation of the regularization method. Moreover, an efficient algorithm based on the finite element method is given. It is shown that the sequence constructed by the algorithm is monotone and converges to the solution of a given free boundary problem.

The last paper deals with numerical approximations for the $m$-membrane problem. We consider minimization of the functional

$$I = \int_\Omega \sum_i^m \left(\tfrac{1}{2}|\nabla u_i|^2 + f_i u_i\right) dx, \tag{2}$$

over the set $\{(u_1, \dots, u_m) \,|\, u_i - g_i \in H_0^1(\Omega),\ u_1 \geq u_2 \geq \cdots \geq u_m\}$.