

The problematic art of counting

Abstract. Many counting problems, like

“In how many ways can a positive integer n be written as a sum of positive integers?”

“Given a polytope P , how many lattice points does the dilated polytope nP contain?”

“How many lines in a $(n + 1)$ -dimensional space meet $2n$ general $(n - 1)$ -planes?”

are solved by finding a closed form for the corresponding *generating function* $\sum_n N_n q^n$, where the N_n are the sought numbers and q is a variable. In this lecture we shall, in addition to the above questions, also address an old problem from enumerative geometry:

“How many plane curves of degree d have r singularities and pass through $\frac{d(d+3)}{2} - r$ given points in the plane?”

In this case the generating function is still unknown, but there has recently been substantial progress on the problem and its generalizations.