

Tangential Derivations, Hilbert Series and Modules over Lie Algebroids

Yohannes Tadesse

Academic dissertation for the Degree of Doctor of Philosophy in Mathematics at Stockholm University to be publicly defended on Friday 28 October 2011 at ~~13:00~~ 14:00 in lecture room 14, house 5, Kräftriket, Roslagsvägen 101.

Abstract

Let A/k be a local commutative algebra over a field k of characteristic 0, and $T_{\{A/k\}}$ be the module of k -linear derivations on A . We study, in two papers, the set of k -linear derivations on A which are tangential to an ideal I of A (preserves I), defining an A -submodule $T_{\{A/k\}}(I)$ of $T_{\{A/k\}}$, which moreover is a k -Lie subalgebra. More generally we consider Lie algebroids g_A over A and modules over g_A .

Paper I: Using the action of an algebraic torus on a monomial ideal in a polynomial ring $A=k[x_1, \dots, x_n]$ we:

- give a new proof of a description of the set of tangential derivations $T_{\{A/k\}}(I)$ along a monomial ideal I , first proven by Brumatti and Simis.
- give a new and direct proof to the fact that the integral closure of a monomial ideal is monomial. We also prove that a derivation which is tangential to a monomial ideal will remain tangential to its integral closure.
- prove that a derivation which is tangential to a monomial ideal is also tangential to any of its associated multiplier ideals.

Paper II: We consider modules M over a Lie algebroid g_A which are of finite type over A . In particular, we study the Hilbert series of the associated graded module of such a module with respect to an ideal of definition.

Our main results are:

- Hilbert's finiteness theorem in invariant theory is shown to hold also for a noetherian graded g_A -algebra S and a noetherian (S, g_A) -graded module which are semisimple over g_A .
- We define a class of local system g_A -modules and prove that the Hilbert series of such a graded module is rational. We also define an ideal of definition for a g_A -module M and prove rationality of the Hilbert series of M with respect to such an ideal.
- We introduce the notion of toral Lie algebroids over a regular noetherian local algebra R and give some properties of modules over such Lie algebroids. In particular, we compute the Hilbert series of submodules of R over a Lie algebroid containing a toral Lie algebroid.

Keywords: *Tangential Derivations, Monomials, Multiplier Ideals, Lie Algebroids, Hilbert series.*

Stockholm 2011

<http://urn.kb.se/resolve?urn=urn:nbn:se:su:diva-62642>

ISBN 978-91-7447-372-8

Department of Mathematics

Stockholm University, 106 91 Stockholm



Stockholm
University