DISPUTATION I MATEMATIK
Christian Lundkvist

skall disputera på avhandlingen

Moduli spaces of zero-dimensional geometric objects

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Abstract of the thesis

The topic of this thesis is the study of moduli spaces of zero-dimensional geometric objects. The thesis consists of three articles each focusing on a particular moduli space.

The first article concerns the *Hilbert scheme* $\mathrm{Hilb}(X)$. This moduli space parametrizes closed subschemes of a fixed ambient scheme X. It has been known implicitly for some time that the Hilbert scheme does not behave well when the scheme X is not separated. The article shows that the separation hypothesis is necessary in the sense that the component $\mathrm{Hilb}^1(X)$ of $\mathrm{Hilb}(X)$ parametrizing subschemes of dimension zero and length 1 does not exist if X is not separated.

Article number two deals with the *Chow scheme* $\mathrm{Chow}_{0,n}(X)$ parametrizing zero-dimensional effective cycles of length n on the given scheme X. There is a related construction, the *Symmetric product* $\mathrm{Sym}^n(X)$, defined as the quotient of the n-fold product $X \times \cdots \times X$ of X by the natural action of the symmetric group $\mathscr{S}_n$ permuting the factors. There is a canonical map $\mathrm{Sym}^n(X) \to \mathrm{Chow}_{0,n}(X)$ that, set-theoretically, maps a tuple $(x_1, \ldots, x_n)$ to the cycle $\sum_{k=1}^n x_k$. In many cases this canonical map is an isomorphism. We explore in this paper some examples where it is not an isomorphism. This will also lead to some results concerning the question whether the symmetric product commutes with base change.

The third article is related to the *Fulton-MacPherson compactification* of the configuration space of points. Here we begin by considering the *configuration space* $F(X,n)$ parametrizing n-tuples of distinct ordered points on a smooth scheme X. The scheme $F(X,n)$ has a compactification $X[n]$ which is obtained from the product X^n by a sequence of blowups. Thus $X[n]$ is itself not defined as a moduli space, but the points on the boundary of $X[n]$ may be interpreted as geometric objects called *stable degenerations.* It is then natural to ask if $X[n]$ can be defined as a moduli space of stable degenerations instead of as a blowup. In the third article we begin work towards an answer to this question in the case where $X = \mathbb{P}^2$. We define a very general moduli stack $\mathscr{X}_{\mathrm{pv2}}$ parametrizing projective schemes whose structure sheaf has vanishing second cohomology. We then use Artin's criteria to show that this stack is algebraic. One may define a stack $\mathcal{SD}_{X,n}$ of stable degenerastions of X and the goal is then to prove algebraicity of the stack $\mathcal{SD}_{X,n}$ by using $\mathscr{X}_{\mathrm{pv2}}$.