Beurling primes and Hardy spaces of Dirichlet series

Athanasios Kouroupis (NTNU, Trondheim, Norway)

An increasing sequence $q = \{q_n\}_{n\geq 1}$, $1 < q_n \to \infty$, such that $\{\log q_n\}_{n\geq 1}$ is linearly independent over $\mathbb{Q}$, called a sequence of Beurling primes. Imitating the construction of the natural numbers through the fundamental theorem of arithmetic we define the set of Beurling integers $\mathbb{N}_q = \{\nu_n\}_{n\geq 1}$, consisting of finite products with factors from q. The corresponding generalized Dirichlet series are of the form $f(s) = \sum_{n\geq 1} a_n \nu_n^{-s}$.

In the classical case, where $\nu_n = n$, Bohr's theorem holds: if f is bounded in a half-plane, then it is uniformly convergent in every smaller half-plane. We will find a sequence of Beurling primes for which both Bohr's theorem and the analogue of Riemann hypothesis are valid. This provides a counterexample to a conjecture of H. Helson concerning outer functions.

In the second part of the talk we will give a necessary condition, in terms of a Nevanlinna-type counting function, for a certain class on composition operators $C_\psi(f) = f \circ \psi$ to be compact on $\mathcal{H}^2$, the Hardy space of Dirichlet series with square summable coefficients. To do that we will add Beurling primes to the structure of the space.

The first part of the talk is joint work with Frederik Broucke and Karl–Mikael Perfekt.