Normalized ground states of nonlinear Schrödinger equations

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We present a simple minimization method to show the existence of normalized ground state solutions to the nonlinear Schrödinger equation

$$\begin{cases} -\Delta u + \lambda u = g(u) & \text{in } \mathbb{R}^N,\ N \geq 3, \\ u \in H^1(\mathbb{R}) \\ \int_{\mathbb{R}^N} |u|^2\,dx = \rho > 0, \end{cases}$$

where ρ is the prescribed mass. Our approach is based on the direct minimization of the energy functional on a suitable constraint. A crucial step is the application of the profile decomposition theorem involving a general Sobolev-subcritical nonlinearity.