Hybrid Conference

Quantum graphs
in
Mathematics, Physics and Applications

QGraph Network Meeting

under **COST Action CA18232**
"Mathematical models for interacting dynamics on networks"

Stockholm University, 8–10 December 2021

Programme

All lectures of the meeting will take place at Stockholm University, Kräftriket 5, room 15, and on Zoom via

`https://stockholmuniversity.zoom.us/j/61034447006`

The schedule refers to Stockholm time (CET).

Wednesday, 8 December

09.00-11.00	Nobel lectures in physics via `https://www.nobelprize.org/`
14:00-14:05	**Opening**
14.05-14.30	D. Mugnolo
14.35-15.00	M. Plümer
15.05-15.30	M. Täufer
15.35-16.00	**Coffee break**
16.00-16.25	J. Kennedy
16.30-16.55	M.-E. Pistol
17.00-17.25	F. Fischer
18.30-	**Conference dinner**

Thursday, 9 December

14.00-14.25	M. Kramar Fijavž
14.30-14.55	G. Sofer
15.00-15.25	V. Pivovarchik
15.30-16.00	**Coffee break**
16.00-16.25	Yu. Latushkin
16.30-16.55	S. Sukhtaiev
17.00-17.25	L. Alon

Friday, 10 December

14.00-14.25	L. Sirko
14.30-14.55	J. Lipovsky
15.00-15.25	J. Kerner
15.30-16.00	**Coffee break**
16.00-16.25	M. Ettehad
16.30-16.55	H. Yu
17.00-17.25	D. Borthwick

The “Trace Space” and genericity statements for metric graphs

Lior Alon (Princeton)

The first genericity result on metric graphs is due to Friedlander 05’, who showed that for any graph structure there is a Baire-generic choice of edge lengths such that all eigenvalues of that metric graph are simple. Berkolaiko and Liu 17’, showed for such a setting, that all eigenfunctions do not vanish on vertices. In this talk, I will introduce the “Trace Space” of a graph structure and a stronger notion of genericity. I will give independent proof for the previous genericity results and will prove in addition that generically the eigenvfunctions do not satisfy any "additional vertex condition". This proof is based on a conjecture of Colin de Verdiere which was recently proved by Kurasov and Sarnak.

Analysis of the heat kernel on a compact metric graph

David Borthwick (Emory University)

We analyze heat kernel associated to the Laplacian on a compact metric graph, with standard Kirchoff-Neumann vertex conditions. An explicit formula for the heat kernel as a sum over loops, developed by Kostrykin, Potthoff, and Schrader, allows for a straightforward analysis of small-time asymptotics. We show that the restriction of the heat kernel to the diagonal satisfies a modified version of the heat equation. This observation leads to an “edge” heat trace formula, expressing the a sum over eigenfunction amplitudes on a single edge as a sum over closed loops containing that edge. Joint work with L. Corsi, E. Harrell, and K. Jones.

On the Spectra of Periodic Elastic Beam Lattices: Single-Layer Graph

Mahmood Ettehad (Minnesota)

We present a full description of spectra for the elastic beam Hamiltonian defined on periodic hexagonal lattices. These continua are constructed out of Euler-Bernoulli beams, each governed by a scalar valued self-adjoint fourth-order operator equipped with a real periodic symmetric potential. Compared to the Schrödinger operator commonly applied in quantum graph literature, here vertex matching conditions encode geometry of the graph by their dependence on angles at which edges are met. We show that for a special equal-angle lattice, known as graphene, dispersion relation has a similar structure as reported for Schrödinger operators on periodic hexagonal lattices. This property is then further utilized to prove the existence of singular Dirac points. We next discuss the role of the potential on reducibility of Fermi surface at uncountably many low-energy levels for this special lattice. Applying perturbation analysis, the developed theory is extended to derive dispersion relation for angle-perturbed Hamiltonian of lattices in a geometric-neighborhood of graphene. In these graphs, unlike graphene, dispersion relation is not splitted into purely energy and quasimomentum dependent terms, but up to some quantifiable accuracy, singular Dirac points exist at the same points as the graphene case. This is a joint work with Burak Hatinoglu.

A Non-Linear Ground State Representation On Graphs

Florian Fischer (Potsdam)

We study energy functionals associated with p-Schrödinger operators on infinite graphs, and develop a non-linear and non-local version of the ground state representation. This representation is an equivalence between functionals, i.e., the energy functional is equivalent to a simplified energy functional consisting of non-negative terms only. We compare this result with its counterpart in the continuum. As applications, we show characterisations of criticality and a Liouville comparison principle. The talk shows work in progress.

Spectral minimal partitions of infinite quantum graphs

James Kennedy (Lisbon)

We study the problem of partitioning unbounded metric graphs based on minimising energy functionals built on the infimum of the spectrum of Dirichlet Laplacian or Schrödinger operators on each of the partition elements. This follows recent works studying similar problems on compact metric graphs.

Remarks on the spectral gap of Schrödinger operators on graphs and domains

Joachim Kerner (Hagen)

We shall present recent results on the spectral gap of Schrödinger operators in different settings. In particular, we are interested in understanding the asymptotic behaviour of the gap in a limit where the volume of the underlying configuration space tends to infinity. It turns out that, depending on properties of the external potential, different scenarios are possible: in some cases the spectral gap behaves as the gap of the free Laplacian and in some cases it does not! This talk is partially based on joint work with M. Täufer (Hagen) and P. Yatsyna (Prague).

Transport equations on non-compact metric graphs

Marjeta Kramar Fijavž (Ljubljana)

We present abstract results on the generation of C_0-semigroups by first order differential operators on the Banach space of L^p-functions on a collections of compact intervals, normalized as $[0, 1]$, and non-compact intervals, parametized along semi-axis $[0, \infty)$, coupled with very general boundary conditions. We then apply these abstract results to study well-posedness of different transport problems on (non-)compact metric graphs.

This is a joint work with Klaus-Jochen Engel (University of L'Aquila).

Asymptotic Perturbation Theory for Extensions of Symmetric Operators

Yuri Latushkin (Missouri)

We will discuss asymptotic perturbation theory for varying self-adjoint extensions of symmetric operators. Employing symplectic formulation of self-adjointness we obtain a new version of Krein formula for resolvent difference, which facilitates asymptotic analysis of resolvent operators via first and second order expansion for the family of Lagrangian planes associated with perturbed operators. Specifically, we derive a Riccati-type differential equation and the first and second order asymptotic expansions for resolvents of self-adjoint extensions determined by smooth one-parameter families of Lagrangian planes. This asymptotic perturbation theory yields a symplectic version of the abstract Kato selection theorem and Hadamard-Rellich- type variational formula for the first and second derivatives of multiple eigenvalue curves bifurcating from an eigenvalue of the unperturbed operator. The latter, in turn, gives a general infinitesimal version of the celebrated formula equating the spectral flow of a path of self-adjoint extensions and the Maslov index of the corresponding path of Lagrangian planes. Illustrations are offered in the context of quantum graphs, periodic Kronig-Penney model, elliptic second order partial differential operators with Robin boundary conditions, etc. Joint work with Selim Sukhtaiev (Auburn University).

Application of quotient graph theory to three-edge star graphs

Jiří Lipovský (Hradec Králové)

We review the quotient graph theory summarized in [1] and apply it to the star graphs with three edges. The quotient graph theory allows decomposing the graphs with symmetry to several smaller, quotient, graphs. The Hamiltonian on the former graph is unitarily equivalent to the orthogonal sum of the quotient graph Hamiltonians. This allows us e.g. to easily compute the secular equation. In our three-edge example we consider standard, δ and preferred-orientation couplings.

This is joint work with V. Ježek based on the paper [2] to appear in Acta Physica Polonica A.

[1] R. Band, G. Berkolaiko, C. H. Joyner, W. Liu, Quotients of finite-dimensional operators by symmetry representations, arXiv preprint, arXiv:1711.00918 [math-ph].

[2] V. Ježek, Jiří Lipovský, Application of quotient graph theory to three-edge star graphs, arXiv preprint, arXiv:2108.05253 [math-ph].

The Ornstein-Uhlenbeck operator on metric star graphs

Delio Mugnolo (Hagen)

We introduce the Ornstein-Uhlenbeck operator on star graphs consisting of finitely many halflines. We first prove that the corresponding evolution equation is well-posed in different function spaces (especially, the classical L^2-space and a Lebesgue space L^2_μ with respect to an appropriate invariant measure) and provide an explicit formula for the solution. We then turn to its spectral theory on L^2_μ, proving in particular that the Ornstein-Uhlenbeck operator has trace class resolvent.

If time allows, we will also show how to use these results to derive an explicit formula for the heat kernel associated with the harmonic oscillator on metric star graphs.

This is joint work with Abdelaziz Rhandi (Salerno).

Generating isospectral but not isomorphic quantum graphs

Mats-Erik Pistol (Lund)

Quantum graphs are defined by having a Laplacian defined on the edges a metric graph with boundary conditions on each vertex such that the resulting operator, L, is self-adjoint. We use Neumann boundary conditions. The spectrum of L does not determine the graph uniquely, that is, there exist non-isomorphic graphs with the same spectra. There are few known examples of pairs of non-isomorphic but isospectral quantum graphs. We have found all pairs of isospectral but non-isomorphic equilateral connected quantum graphs with at most seven vertices. We find three isospectral triplets including one involving a loop. We also present a combinatorial method to generate arbitrarily large sets of isospectral graphs and give an example of an isospectral set of four. This has been done this using computer algebra and we will give a demonstration if time allows.

Cospectral quantum graphs

Vyacheslav Pivovarchik (South Ukrainian National Pedagogical University)

Spectral problems are considered generated by the Sturm-Liouville equation on connected simple equilateral graphs with the Neumann and Dirichlet boundary conditions at the pendant vertices and continuity and Kirchhoff's conditions at the interior vertices. We highlight the cases where the first and the second terms of the asymptotics of the eigenvalues uniquely determine the shape of the graph or of its interior subgraph.

Torsional rigidity of compact quantum graphs

Marvin Plümer (Hagen)

In this talk, we give an introduction to the theory of torsional rigidity of Dirichlet Laplacians on metric graphs. Based on a variational description going back to Pólya, we provide surgery principles for the torsional rigidity that, in turn, will be used to prove isoperimetric inequalities for the torsional rigidity. Finally we prove Kohler–Jobin-type spectral estimates for the Dirchlet Laplacian in terms of the torsional rigidity. Such estimates are particularly interesting, since we will show that the computation of the torsional rigidity can be reduced to the solution of a finite-dimensional linear system, whereas the computation of the corresponding eigenvalues usually leads to nonlinear system.

Isoscattering chains of graphs and microwave networks

Leszek Sirko (IFPAN)

We study a problem that generalizes a famous question of Mark Kac "Can one hear the shape of a drum?" [1-3] to the infinite chains of isoscattering open graphs and networks with preserved scattering properties coupled to the dissipative environment [4]. In an experimental and mathematical approach our work goes beyond prior results. We describe chains of isoscattering graphs and microwave networks and demonstrate that using a trace function one can address the unsettled until now problem of whether scattering properties of open complex graphs and networks are uniquely connected to their shapes? The question is answered in negative. Joint work with Małgorzata Białous, Adam Sawicki, and Michał Ławniczak.

Acknowledgements: This work was supported in part by the National Science Centre, Poland, Grant No. UMO-2018/30/Q/ST2/00324.

[1] M. Kac, Am. Math. Mon. 73, 1 (1966).

[2] O. Hul, M. Ławniczak, S. Bauch, A. Sawicki, M. Kuś, and L. Sirko, Phys. Rev. Lett. 109, 040402 (2012).

[3] M. Ławniczak, A. Sawicki, S. Bauch, M. Kuś, and L. Sirko, Phys. Rev. E 89, 032911 (2014).

[4] M. Ławniczak, A. Sawicki, M. Białous, and L. Sirko, Scientific Reports 11, 1575 (2021).

Differences between Robin and Neumann eigenvalues on metric graphs

Gilad Sofer (Technion)

Given a metric graph with Neumann-Kirchhoff condition at all vertices, one may consider a perturbation given by placing a δvertex condition at a selected subset of vertices. This results in an increase of the Laplacian eigenvalues. The sequence of differences between the eigenvalues of the perturbed and original operators, which is known as the Robin-Neumann gap, has been recently studied in the context of Planar domains ([2]) and star graphs ([1]). We study the prop erties of the Robin-Neumann gap on general metric graphs and compare it to the results known or conjectured for planar domains. In particular, we make the connection to inverse spectral problems, by showing that the mean value of the Robin-Neumann gap is determined by the total length of the graph, the strength of the perturbation, and the degrees of the perturbed vertices. The talk is based on a joint work in progress with Ram Band, Holger Schanz and Uzy Smilansky.

References

[1] Gabriel Riviére and Julien Royer. Spectrum of a non-selfadjoint quantum star graph. J.Phys.A, 53(49):495202, 2020.

[2] Zeev Rudnick, Igor Wigman, and Nadav Yesha. Differences between Robin and Neumann eigenvalues. arXiv:2008.07400.

Two classes of dynamically defined quantum graphs

Sukhtaiev, Selim (Auburn University)

In this talk, we will discuss two classes of dynamically defined quantum graphs exhibiting interesting spectral behavior. The first part of the talk concerns spectral and dynamical localization for Anderson-type models on trees and random Hamiltonians with general point interactions. In the second part, we will discuss a class of dynamically defined anti-trees exhibiting singular continuous spectrum. This talk is based on several joint projects with D. Damanik, L. Fang, J. Fillman, M. Helman, J. Kesten.

When are generic eigenfunctions actually universal?

Matthias Täufer (Hagen)

So-called "generic eigenvalues" of quantum graphs are single eigenvalues with eigenfunctions that vanish at no vertex (except of course for Dirichlet vertices if the graph has any). They are called "generic" because generically among all metric graphs with a given topology, one will encounter an infinite sequence of generic eigenvalues. They are useful to have for various applications. In this talk we will show that for tree graphs and connected graphs with at least one Dirichlet vertex, generic eigenfunctions are actually universal: They always exist and there are no exceptions. Furthermore, this is sharp since beyond this class, one can always find exceptions. The talk is based on joint work with Marvin Plümer.

Application of the Davies Inequality

Haozhe Yu (Emory University)

We focus on the application of the Davies inequality on quantum graphs. Although the Davies inequality is a well known inequality on manifolds, as far as we know it has not been applied on quantum graphs. We will prove that the Davies inequality is available on quantum graphs and construct upper bounds of eigenvalues using the inequality. We will also compare our estimate with the estimate using Betti number in the paper by Berkolaiko, Kennedy, Kurasov and Mugnolo on certain graphs.

This is a work in progress with Dr. Borthwick, Dr. Harrell, and Kenny.