Workshop i dynamiska system

KTH 8-9 maj 2014

Abstracts

Magnus Aspenberg, Lund

Mating polynomials in degree 2 *and* 3

Abstract: The idea of mating was invented by Douady and Hubbard in the 1980s as a way to partially parameterize the space of rational functions of a given degree $d \geq 2$ by pairs of polynomials. In the quadratic case, take two polynomials $f_1(z) = z^2 + c_1$ and $f_2(z) = z^2 + c_2$ where c_1 and c_2 do not lie in conjugate limbs of the Mandelbrot set. Let their Julia sets be J_1 and J_2 respectively and their filled in Julia sets K_1 and K_2. If these sets are locally connected and connected then it is possible to glue the filled in Julia sets along the boundaries in reverse order. The resulting set is under good circumstances the (homeomorphic copy of a) Julia set of a rational map of degree 2. Higher degree matings also exist. For instance, Newton maps of degree 3 arise often as matings of cubic polynomials. I will outline same basic ideas about the mating-construction and pose some results and open questions.

Michael Björklund, Göteborg

Products, dynamics and harmonic functions

Abstract: It is often useful, when studying iterations of a fixed continuous transformation on a compact space, to consider the invariant (probability) measures for the transformation. The (automatic) existence of an ergodic probability measure for the transformation tends to give a rich supply of points whose "orbit statistics" can be well understood.

When faced with the task of understanding the "joint orbit statistics" of a given collection of transformations on a compact space, one is often forced to conclude that there are no common invariant measures for ALL of the transformations, and that the invariant measures for the individual transformations can be very singular to each other.

The aim of this talk is to discuss various substitutes for invariant measures in these situations and what kind of dynamical information one can extract from these measures.

We shall be mostly concerned with applications relating to the geometry and combinatorics of large infinite subsets of countable groups.

Neil Dobbs, Helsinki

Free energy jumps up

Abstract: We shall discuss the thermodynamic formalism for smooth maps of the unit interval. Given a potential function, the free energy of an invariant probability measure can be defined, and the pressure is the supremum of the free energy. Equilibrium measures are ones which realise the supremum. Since free energy does not even depend semi-continuously on the measure, existence and continuity properties of equilibrium measures are not immediate. We will explain why, for interesting potentials, the free energy is almost semi-continuous, from which we obtain existence and statistical stability of equilibrium states, under a positive entropy assumption. Joint with Mike Todd.

Peter Hazard, Uppsala/Toronto

Hénon-like Maps: Combinatorics, Braids and Renormalisation.

Abstract: I will discuss recent joint work by myself, Andre de Carvalho and Toby Hall on Braid Equivalence of Hénon-like Maps. In the case of real unimodal maps, an irreducible combinatorial type determines uniquely a renormalisation operator. Recently, these operators were extended to a more general space of maps – Henon-like maps – by de Carvalho, Lyubich, Martens and myself. We consider when two unimodal combinatorial types, which give distinct unimodal renormalisation operators, give 'the same' Hénon renormalisation operator. Time permitting we will also discuss how these combinatorial types fit together in parameter space, via a new operator.

Anders Karlsson, Uppsala/Geneva

A limit law for products of random surface homeomorphisms

Abstract: Thurston established a spectral theorem for individual surface homeomorphisms, analogous to the Jordan normal form for matrices. I will discuss extensions of this to products of random surface homeomorphisms.

Hans Lundmark, Linköping

Dynamics of peaked solitons

Abstract: The Camassa-Holm shallow water wave equation from 1993 has many peculiar properties, one of which is that it admits nonsmooth multisoliton solutions with peaked wave crests. The dynamics of these so-called peakon solutions are governed by a finite-dimensional Hamiltonian system with the locations and amplitudes of the individual solitons as canonical position and momentum variables. This system is completely integrable, and in fact explicitly solvable in terms of elementary functions; the derivation of the exact solution formulas (and the study of their properties) involves some classical 19th century

mathematics related to orthogonal polynomials. There are also other integrable PDEs with explicitly computable peakon solutions, such as the Degasperis-Procesi equation (1998) and Novikov's eqation (2008), where the integration of the governing dynamical system is more complicated and leads to interesting generalizations of the classical concepts. In my talk, I will try to give an overview of this topic, and also mention some new developments. For example, it has been found recently that the interaction between peakons and antipeakons (with negative elevation) in Novikov's equation can display some very intricate dynamics not seen for the other equations.

Elio Mazzeo, Uppsala/Toronto

Circle Maps, Renormalization, and Rigidity Theory

Abstract: The talk will discuss the dynamics of circle maps (orientation-preserving circle homeomorphisms (o.p.c.h)).

We will review the topological classification of these circle maps, namely the Poincare Classification and the Denjoy Theory. We will introduce the beautiful construction of the dynamical partition of the circle. We will review the connection between the concept of the Diophantine properties of the irrational rotation number of our map and the smooth classification of (sufficiently regular) circle maps (the so called "rigidity" theory), in the context of three important classes: diffeomorphisms, critical maps, and maps with a break point.

For circle maps with a break point, we will discuss two results that were obtained in the rigidity theory of circle maps with a break point, as part of my dissertation.

The first main result is a proof that C^1 rigidity holds for circle maps with a break point for almost all irrational rotation numbers. This result is joint work with Kostya Khanin and Sasa Kocic.

The second main result has to do with the family of fractional linear transformation (FLT) pairs. An FLT-pair T is a circle homeomorphism that consists of two branches each of which is an FLT. Such a map can be viewed as a circle map with two neighbouring break points lying on the same orbit. For this family, C^1 rigidity holds for all irrational rotation numbers without any restriction (the so-called "robust rigidity").

Tomas Persson, Lund

Limsup-sets of random covers

Abstract: Suppose that we have a sequence of open subsets of a torus. We translate these open sets randomly and form the limsup-set, that is the set of points that are covered infinitely often by the translated open sets. I will talk about fractal properties that hold almost surely for such limsup-sets, and how such properties can be proved in a simple way using a certain lemma.

Andreas Strömbergsson , Uppsala

On the probability of a random lattice avoiding a given set

Abstract: Let X be the space of lattices of covolume one in d-dimensional Euclidean space $\mathbb{R}^d$, equipped with its invariant probability measure. Let $p(C)$ be the probability that a random lattice L in X is disjoint from a given set C in $\mathbb{R}^d$. For certain specific choices of C, this probability $p(C)$, as well as a variant where one conditions on L containing a given point p on the boundary of C, have appeared as limit functions in several asymptotic problems in number theory and mathematical physics. In particular I will describe how this probability appears as the transition kernel for a Markov chain which describes the periodic Lorentz gas in the Boltzmann-Grad limit. I will discuss some fundamental properties of this probability $p(C)$; especially its behavior for C large. Among the applications is a recent proof by Marklof and Toth of a superdiffusive central limit theorem for the displacement of a test particle in the limit of large times and low scatterer densities for the periodic Lorentz gas.

The talk is partly based on joint work with Jens Marklof.

Mircea Voda, Uppsala/Toronto

Effective Estimates for the Lyapunov Exponents of the Anderson Model on a Strip

Abstract: I will discuss some recent results on explicit lower bounds for the non-negative Lyapunov exponents associated with random Schroedinger operators on strips in $\mathbb{Z}^2$. The talk is based on joint work with Ilia Binder and Michael Goldstein.